The Medical Expansion, Life Expectancy, and Endogenous Directed Technical Change*

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Abstract

We build a unified quantitative theory of rising adult life expectancy and income growth over the past two centuries, and the emergence of a modern health sector in the 20th century. We interpret the data as three phases of a dynamic equilibrium in which households are initially poor, the price of health goods is prohibitively high, and life expectancy is stagnant. As technological progress fuels income growth, households begin consuming basic health goods and life expectancy rises in the 19th century. A century later, directed technological progress leads to the emergence of a modern health sector. In the model, the quality-adjusted relative price of modern health goods declines by 2.5% annually between 1940 and 2020. Counterfactual analyses suggest that during this period one-fourth of life expectancy gains reflect modern health innovations, and that public health R&D during World War II played a key role in launching the modern health sector.

JEL Codes: E13, O41, I15

Keywords: Life Expectancy, Modern Health Sector, Endogenous and Directed Technological Change, Transition

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1 Introduction

Over the last two centuries, the United States transitioned from an economy with short and stagnant adult lives and negligible medical spending to one that is an order of magnitude richer, with long lives and a modern, innovation-intensive health sector. Remaining life expectancy of a twenty-year-old person has risen from approximately 40 years in 1820 to above 60 years by the eve of the COVID-19 pandemic, while real per capita income has increased from $2,674 to $55,355 (in 2011 dollars) and close to 20% of that income is now spent on goods produced by a modern, high-tech health sector.[1]

In this paper, we propose a quantitative theory of the joint dynamics of income growth and demand for longevity to explain these observations, namely the timing and magnitude of the rise in adult life expectancy since the early nineteenth century, and the emergence of a high-tech health sector in the mid-twentieth century whose output commands an increasing share of expenditure and employment. We provide empirical evidence on adult life expectancy since the early nineteenth century to discipline the theory, and then use it to investigate how much of the increase in adult life expectancy can be attributed to the expansion of the modern health sector, and how important public health policies have been for its emergence and expansion.

Our model is a two-sector OLG economy with endogenous and directed technical change in a consumption sector and a modern health goods sector in which income growth, life expectancy, and the quality and relative price of modern health goods are jointly determined in general equilibrium. On the demand side, our theory combines non-homothetic demand for longevity with the assumption that households can improve their longevity by investing in basic health goods produced by the consumption goods sector, capturing for example basic food and hygiene, and by investing in modern health goods, capturing modern medical treatments. On the supply side, final consumption and modern health goods rely on intermediate inputs whose qualities evolve endogenously through R&D investments by entrepreneurs who are incentivized by the prospect of profits and respond to changes in the relative price and demand for health goods. The model interprets the facts as three phases of a dynamic equilibrium in which households are initially poor and the price of health goods is prohibitively high so that households optimally spend nothing on longevity, life is short and life expectancy is stagnant. As income grows, fueled by technological progress, households start consuming basic health goods, life expectancy

[1] Real income pc in constant 2011 US dollars is from the Maddison data, Bolt and van Zanden (2024).

rises, and directed technological progress eventually, with a delay of approximately 100 years, leads to the emergence and expansion of a modern health sector.

We discipline the model parameters governing household demand using long-run U.S. evidence on income per capita, life expectancy, and the rise in aggregate modern health expenditures. To do so, we construct estimates of life expectancy for the U.S., combining historical age-specific mortality data from Haines (1994) and Hacker (2010), the HLD and HMD. We focus on remaining life expectancy at age 20 consistent with our model of adult health investments, and we focus on the cohort concept of life expectancy to capture the role of medical innovation over an individual's life-cycle. The model accounts well for the time series in per capita income, life expectancy, and spending on the modern health sector since its emergence.

We first turn to the model-implied evolution of the relative price of health goods to validate whether the model's decomposition of the observed and targeted rise in health spending into untargeted relative quality and price movements is plausible. In particular, the model provides a disciplined interpretation for the role of relative quality improvements for the observed rise in relative medical prices in postwar U.S. data because it allows us to distinguish between the price per unit of quality-adjusted health care and the quality-unadjusted price of a fixed medical service. Through the lens of the model, the quality-unadjusted relative price of health goods has risen almost threefold since 1940, closely matching the BEA health price index. Once we properly adjust for relative quality improvements in modern health goods, the model implies that the price of health goods has fallen by about 2.5% per year relative to consumption goods since 1940, in line with existing estimates of the effective price of different health outcomes, and we find that the decline in the price of medical care has slowed over time and will continue to do so, driven by slowing relative productivity growth in the health sector.

We then use the calibrated model as a quantitative laboratory to answer two applied questions. We first ask what share of the increase in life expectancy from 1940 onward can be attributed to the modern health sector. When we recompute life expectancy by simply rendering spending on modern health goods ineffective without allowing households to reoptimize, we find that life expectancy falls almost 10 years in 2020 and postwar life expectancy gains fall by approximately 80%. However, leveraging our structural model, we then ask what life expectancy would be in a counterfactual economy in which the modern health sector never emerges, our preferred measure of the contribution of modern health. Households reoptimize their health expenditures, raise investment into basic health, and the share of postwar life expectancy gains attributed to the modern health sector falls to approximately one fourth, driven by the fact that basic and modern health goods are relatively substitutable. This suggests that both the modern health sector, but even more so income-growth-induced increases in spending on basic health (e.g., better food and hygiene) played a major role in the expansion of life expectancy into the 21st century.[2]

Second, we use the model to analyze the role of public health expenditure policies for the emergence and expansion of the modern health sector. Specifically, we investigate the roles played by U.S. government spending on R&D of modern health treatments during World War II as documented, e.g., by Gross and Sampat (2025)[3], as well as the emergence of Medicare and Medicaid during the 1960s. Through the lens of the model, while both programs ultimately spur health innovation and reduce the relative health price, they also reduce net household income due to being tax financed, leaving the overall effect on life expectancy and welfare ambiguous. We find that the war-related R&D policies, modeled as a one-time R&D subsidy in 1940, had significant positive effects on life expectancy and welfare for subsequent cohorts. Without the subsidy the modern health sector would have barely emerged in 1940 and initial health prices would have been twice as high, such that even the 1940 cohort that financed the subsidy experienced a small positive effect on life expectancy and welfare from the program. Through its persistent effect on productivity, the 1940 R&D subsidy accounts for almost one year of life expectancy and generates sizable welfare benefits by 2020. Similarly, the expansion of public health spending on the old through Medicare and the nursing home component of Medicaid has been a key driver of demand for modern health goods, without which the modern health sector would account for about one third less of overall spending in 2020. However, we find its general equilibrium effects on households, working through income and the relative modern health price, to be negligible with small negative welfare effects for the 2020 cohort.

Related Literature. Our model has three key building blocks, a two-period overlapping generations structure with production akin to Diamond (1965), endogenous investments into health and longevity by private households, as in Grossman (1972) and the endogenous evolution of technological change in the Schumpeterian growth tradition (see, e.g., Aghion

[2]This finding is consistent with Fogel (1993) who argues that improvements in hygiene and diet are the main drivers of the increase in life expectancy even in the presence of modern medicine.

[3]Here, we refer to the Office of Scientific Research and Development (OSRD) and particularly the Committee on Medical Research (CMR), which was subsumed by the NIH after the war. These programs had the goal to test and produce technologies and medical treatments to help the war effort, which included mass-produced penicillin and malaria treatment.

and Howitt 1992 or Aghion and Howitt 1998), whose speed differs across sectors in the economy, akin to Acemoglu and Guerrieri (2008). It seeks to describe the path of economic and health stagnation, take-off during a transition period and, eventually, balanced growth as one dynamic equilibrium, as in the general literature on unified growth theory (see, e.g., Galor 2011 or Hansen and Prescott 2002).

In trying to explain long-run trends in life expectancy and connect it to technological progress our paper builds on the work by Cervellati and Sunde (2005) who develop a model of the take-off of life expectancy by modeling the feedback loop between income growth, human capital formation, increases in life expectancy and the size of the population. In contrast to them, we seek to provide a unified theory not only of the take-off in life expectancy in the 19th century, but also the emergence of the modern health sector. That purpose is shared with Hejkal, Ravikumar, and Vandenbroucke (2024) but their focus is on explaining cross-country differences (and similarities) in the reduction of mortality as well as the evolution of the world population.

More broadly, in terms of model-building this paper contributes to the literature on health spending, R&D in the health sector and endogenous growth. Borger, Rutherford, and Won (2008) develop a model with endogenous technology adaptation in the health sector to predict future health spending shares and conclude that health spending will slow down. Ehrlich and Yin (2013) construct an endogenous growth model where human capital is the engine of growth. Both these elements are encompassed in the work of Kuhn and Prettner (2016) who model a final goods sector with an intermediate R&D sector and a labor-intensive health sector. They argue that an expansion of this sector may reduce growth by shifting resources away from R&D spending. Like us, Frankovic and Kuhn (2023) develop an overlapping generations model with endogenous health and two production sectors to evaluate the quantitative impact of the introduction of health insurance through Medicare and Medicaid on health spending trends, macroeconomic performance, and trends in life expectancy since 1965 when Medicaid was introduced. In their model, the growth rate of the final goods sector is exogenous and endogenous in the health sector (as in the work by Böhm, Grossmann, and Strulik 2021 who model the evolution of individual health through an accumulation of health deficits). In contrast, we model endogenous growth symmetrically in both sectors, but permit it to be unbalanced.[4]

[4]An important mechanism in these models is a market size effect which triggers innovation spending. Empirical support for this mechanism in the health sector is provided by Acemoglu and Linn (2004).

Finally, a vibrant positive literature studies potential reasons behind the increase in the health expenditure share in U.S. postwar data and a normative literature explores what share of economic activity should be dedicated to health goods and services. The normative perspective includes the work by Hall and Jones (2007), Jones (2004) and Jones (2016). Hall and Jones (2007) model health spending as a superior good with an income elasticity larger than one. As a consequence, income growth leads to an expansion of the health expenditure share, as in our paper. We extend this framework to a two-sector model with a symmetric treatment of endogenous growth in both sectors, and where unbalanced growth emerges as an equilibrium outcome. In contrast to their paper our main purpose is positive, seeking to understand the expansion of the modern medical sector. Jones (2004) develops an endogenous growth model with R&D to explain the increasing health expenditure share. Our model shares the same broad narrative, but provides an explicit treatment of production in a two-sector model so that income growth spurs quality improvements and thus technological progress in both sectors. Jones (2016) also considers growth in two sectors by studying the optimal rate of consumption growth relative to growth of life-saving technologies.

Seeking to understand the long-run evolution of life expectancy and medical spending, we seek to contribute to the literature on the drivers of aggregate health expenditures. Anderson, Reinhardt, and Hussey (2003) argue that the increase of the health spending share is mainly due to the relative price increase of medical goods.[5] This may be due to an increase in market power of the supply side relative to the demand side in the market for health goods.[6] Alternatively, it could result from imperfectly measured quality improvements as our model implies.[7] Quality improvements in turn are the result of costly technological progress, which according to Fonseca, Michaud, Galama, and Kapteyn (2021) contribute to about 50 percent of the observed expenditure increases. Finally, we

[5] Fonseca, Langot, Michaud, and Sopraseuth (2023) estimate US health prices to be 33% higher than those of European countries, which explains 60% of differences in health expenditures.

[6] A growing literature attributes increases in health spending in the US and cross-country differences across the OECD to health prices and inefficiencies in health care markets, see, e.g. Cooper, Craig, Gaynor, and Reenen (2019), Horenstein and Santos (2019), and Pretnar and Feldman (2025). Our monopolistically competitive pricing structure in the health sector may reflect such inefficiencies. Higher prices, and the associated higher average asset returns could also be a reflection of compensation for medical innovation risk, see e.g., Koijen, Philipson, and Uhlig (2016).

[7] In this regard, our paper relates to the literature on a lack of quality adjustments of health goods, see, e.g., Cutler, Ghosh, Messer, Raghunathan, Rosen, and Stewart (2022) and Dunn, Hall, and Dauda (2022). In fact, our model predicts a reduction of a quality adjusted price index in line with this literature.

share with Zhao (2014) the perspective that part of the increase in health expenditures is attributable to the introduction and expansion of public social insurance programs.[8]

Section 2 presents stylized facts on life expectancy, aggregate health spending and prices of health goods used to motivate, calibrate and evaluate the model. Section 3 lays out the model and defines the equilibrium. Section 4 contains a theoretical characterization of parts of the equilibrium and Section 5 presents the calibration of the model. Section 6 contains the quantitative evaluation of the model, both along dimensions targeted in the calibration and along dimensions not targeted in the calibration. Section 7 concludes.[9]

2 Stylized Facts

Life Expectancy When documenting life expectancy for the last two centuries we face two crucial choices. First, in the early period a significant increase of life expectancy is due to a decline in child mortality, with later improvements mainly accruing to increased life expectancy conditional on survival to adulthood. Since our model focuses on the second part, the improved longevity of adults, so will our discussion of the data. Second, life expectancy at a given point in time can be measured using purely cross-sectional survival rates or using cohort survival rates. The first, cross-sectional concept only requires age-specific survival rates at a given point in time, but assumes that a current 20 year old adult will have the same survival rate at age 50 (that is, 30 years into the future) as a current 50 year old individual, thereby ignoring potential technological improvements in the health sector. Cohort life expectancy is more data-demanding since it requires future age-specific survival rates of the cohort under consideration. Since it fully captures the impact of medical innovations, a key aspect of our model, we prefer this concept for the purpose of this paper. In conclusion, our main focus in the quantitative analysis will be on remaining cohort life expectancy at age 20.

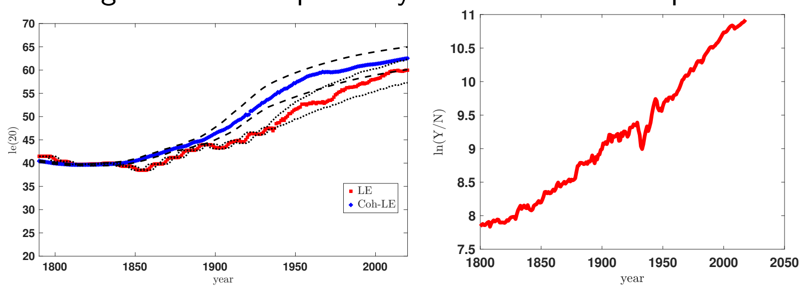
Panel (a) of Figure 1 shows remaining life expectancy at age 20 according to the cross-sectional and cohort concept. We look at remaining (cohort) life expectancy at age 20 of a person in a given year, and make the following observations. First, before about 1840, remaining cohort life expectancy in the US was basically flat and the average—taken for the years 1790 to 1840—was about 39.83 years. Second, since then life expectancy has been increasing so that now—i.e., in year 2020—remaining cohort life expectancy at

[8]A parallel literature studies the sources of level differences in health spending shares across countries and attributes the higher share in the U.S. to the fact that the U.S. is the leading country for the invention of new costly health products, see Chandra and Skinner (2012).

[9]Details of the facts and the theoretical derivations and calibration of the model are in the Appendix.

age 20 stands at about 62.5 years. Acknowledging the uncertainty about the very precise timing of the take-off in adult life expectancy—which we illustrate here by displaying the bootstrapped 95% confidence intervals of cohort life expectancy—, we date this take-off at 1840 for the remainder of this paper.

Figure 1: Life Expectancy and Income Per Capita



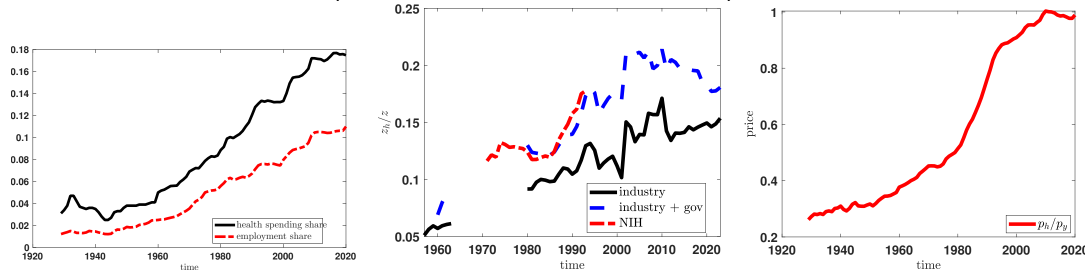
(a) Life Expectancy (b) GDP per Capita (Log Scale)

Notes: Panel (a): Cross-sectional life expectancy and cohort life expectancy. 95% Bootstrapped confidence intervals are shown as dashed lines. *Sources*: Haines (1994), Hacker (2010), HLD, HMD, own computations, see Appendix A for details. Panel (b): Log Income per Capita in the U.S., 1800-2018. *Source*: Maddison Project Data Base, Bolt and van Zanden (2024).

Income per Capita over the Last Two Centuries Our theory ascribes crucial importance to the increase in income per capita in generating the take-off, first in life expectancy and then in the emergence of a modern health sector, driven by rising demand for health goods from the household sector. We therefore briefly review the main facts concerning income per capita (growth) in the U.S. over the long run. Accordingly, Panel (b) of Figure 1, which plots the natural logarithm of real income per capita for the U.S. for the last two centuries, as documented in the Maddison Project Data Base (Bolt and van Zanden 2024), displays the well-documented fact that income per capita growth started to rise around the turn of the 19th century and income per capita has grown at a roughly constant (in fact slightly accelerating) pace of approximately 1.5% per year since 1820.

The Emergence and Evolution of the Modern Health Sector The third set of facts that motivate our analysis is the emergence and growth of a modern health sector in the 20th century. Parker (2019) dates the start of the era of modern medicine at ca. 1920, pointing to landmark breakthroughs such as the discovery of penicillin in 1928 and its mass production towards the end of World War II, the analysis of blood types and blood transfusion and the associated understanding of the causes of diabetes, as well as the emergence of cancer research.

Figure 2: Health Shares (Spending, Employment, R&D) and Relative Health Price



(a) Spending and Employment (b) R&D Spending Share (c) Relative Price

Notes: Panel (a): Total nominal national health expenditure from the Centers for Medicare & Medicaid Services (CMS) and Gibson, Levit, Lazenby, and Waldo (1984) relative to nominal GDP is from BEA data. Year 2020 value is average of 2019 and 2021 values. The employment share is computed as full-time equivalent employees in the health sector relative to the total number of US full-time equivalent employees from BEA Tables 6.4 and 6.5. Panel (b): The nominal R&D expenditure share data are likewise defined as the ratio of (the sum of) nominal health R&D expenditures by industry and the government relative to corresponding total nominal R&D expenditures. Data are from Jones (2016), the National Center for Science and Engineering Statistics for the period 1997-2023, and the National Health Expenditures database of the CMS for 1960-2023. Panel (c): The relative health price is computed as the ratio of the price indices of household expenditures on health services to the GDP price index from BEA Table 1.6.4.

In Panel (a) of Figure 2 we plot health spending and employment shares and in Panel (b) the R&D investment shares devoted to the health sector. The share of economic activity contributed by the health sector was low and stable prior to World War II and since then has been steadily increasing over time to close to 20% of overall household spending and of R&D investment and to about 11% of overall employment. It is this emergence and continued expansion of the modern health sector we seek to explain as the endogenous equilibrium outcome driven by income growth and government intervention on the demand side, and endogenous (temporally) unbalanced technological progress on the supply side.[10] Panel (c) shows the relative price of health goods, measured as ratio of the price index of household expenditures on health services to the GDP deflator. Health goods have become more expensive over time; it is an open question what share of this increase can be attributed to quality improvements in modern health care. Our model permits the interpretation of this observation since it delivers a time series for the price of one unit of health care, and one for the price of one unit of quality-adjusted health goods.

[10] As discussed in Section 1, the US government heavily subsidized science and technology spending on R&D of modern health during World War II. In contrast, US Medicare was only introduced in 1965. This supports our interpretation of the data that emergence and growth of the health sector were primarily the consequence of research and development efforts, which in turn are triggered by economic developments, rather than interpreting the introduction of Medicare as the trigger as in Frankovic and Kuhn (2023).

3 The Model

The model is a small open OLG economy; in each period t a unit mass of identical young individuals is born who live for up to two periods. We denote the endogenous share of individuals reaching old age by n_t^o and the total population by $n_t = 1 + n_t^o$. When young, households work, earn income, spend resources on health and save, and consume their savings in the second period of life. The production side has two sectors, $j \in \{c, h\}$, where c denotes the basic consumption sector and h the modern health sector. In both sectors, production consists of: (i) final goods producing firms operating under perfect competition; (ii) monopolistically competitive suppliers of intermediate goods who sell differentiated varieties to final goods producers at a markup; and (iii) R&D firms who seek to invent intermediates of higher quality, incentivized by these markups, and who, if successful, become the monopolistically competitive suppliers of a specific intermediate.

3.1 Firms, Production and R&D

Final Goods Producers In each sector a representative firm uses a continuum of intermediate inputs indexed by i and labor to produce output y_{jt} according to

$$y_{jt} = \left(\int_0^1 q_{jit}^{1-\alpha_j} y_{jit}^{\alpha_j} di \right) l_{jt}^{1-\alpha_j}, \tag{1}$$

where $0 < \alpha_j < 1$. l_{jt} is labor employed in sector j, and y_{jit} and q_{jit} denote the quantity and quality of intermediate input i used in sector j, respectively. Denote the sector-j output price by p_{jt}, the wage by w_{jt}, and the price of one unit of intermediate good i by p_{jit}. Since final-goods firms in each sector behave competitively and take all prices and qualities as given, she chooses labor and intermediate inputs to maximize

$$\pi_{jt} = \max_{l_{jt}, (y_{jit})_{i \in [0,1]}} p_{jt} \left(\int_0^1 q_{jit}^{1-\alpha_j} y_{jit}^{\alpha_j} di \right) l_{jt}^{1-\alpha_j} - w_{jt} l_{jt} - \int_0^1 p_{jit} y_{jit} di. \tag{2}$$

with optimality conditions for labor l_{jt} and intermediates y_{jit}

$$p_{jt} (1 - \alpha_j) \left(\int_0^1 q_{jit}^{1-\alpha_j} y_{jit}^{\alpha_j} di \right) l_{jt}^{-\alpha_j} = w_{jt}, \tag{3}$$

$$p_{jt} \alpha_j q_{jit}^{1-\alpha_j} y_{jit}^{\alpha_j - 1} l_{jt}^{1-\alpha_j} = p_{jit}. \tag{4}$$

We choose the basic consumption good as numeraire and normalize p_{ct} to 1 and denote by $p_t = p_{ht}/p_{ct} = p_{ht}$ the relative price of goods produced by the modern health sector.

Intermediate Goods Producers Each intermediate good producer i is a monopolist that uses capital to produce the intermediate good according to:

$$y_{jit} = k_{jit}. \tag{5}$$

We assume a small open economy with exogenous, constant gross rental rate of capital R, and capital depreciates fully after use, so that $R = 1 + r$, where r is the simple return on capital. Intermediates producers choose capital input k_{jit} and price p_{jit} to maximize

$$\pi_{jit} = \max_{p_{jit}, y_{jit}} \left\{ p_{jit} y_{jit} - R y_{jit} \right\}, \tag{6}$$

taking demand from the final goods producer as given. The first order condition reads as

$$y_{jit} = \left(\frac{p_{jt} \alpha_j^2}{R} \right)^{\frac{1}{1-\alpha_j}} q_{jit} l_{jt}, \tag{7}$$

which, combined with (4), delivers the standard markup over marginal cost R, and profits

$$p_{jit} = \frac{R}{\alpha_j} > R \quad \text{and} \quad \pi_{jit} = \frac{1-\alpha_j}{\alpha_j} R y_{jit} > 0. \tag{8}$$

Both y_{jit} and π_{jit} are linear in quality (see (7) and (8)) permitting aggregation by sector.

Aggregation Define the aggregate quality index of intermediate inputs in sector j as

$$q_{jt} \equiv \int_0^1 q_{jit} di. \tag{9}$$

Then aggregation for each production sector $j \in \{c, h\}$ immediately follows:

Proposition 1. *In each sector j, aggregate capital, output and profits are only functions of q_{jt}, rather than a function of the entire distribution of the $\{q_{jit}\}_i$ and are given by*

$$k_{jt} = \int_0^1 k_{jit} di = \int_0^1 y_{jit} di = \left(\frac{p_{jt} \alpha_j^2}{R} \right)^{\frac{1}{1-\alpha_j}} q_{jt} l_{jt} \tag{10a}$$

$$y_{jt} = k_{jt}^{\alpha_j} \left(q_{jt} l_{jt} \right)^{1-\alpha_j} \tag{10b}$$

$$\pi_{jt} = \int \pi_{jit} di = \alpha_j \left(1 - \alpha_j \right) p_{jt} y_{jt}. \tag{10c}$$

The factor prices in each sector j satisfy

$$w_{jt} = (1 - \alpha_j) \frac{p_{jt} y_{jt}}{l_{jt}} \tag{11a}$$

$$R = \alpha_j^2 \frac{p_{jt} y_{jt}}{k_{jt}}. \tag{11b}$$

In each sector j factor input payments plus profits exhaust the value of output:

$$p_{jt} y_{jt} = \pi_{jt} + R k_{jt} + w_{jt} l_{jt}. \tag{12}$$

Proof. Follows directly from the production function (1) and the optimality conditions of final and intermediate goods producers. Appendix B.1 contains the derivations. □

The proposition states that in each sector output is produced with a Cobb-Douglas production function with capital and labor as inputs, with a technology level given by q_{jt}. However, since final goods producers cannot rent capital directly and have to go through monopolistically competitive intermediaries, owners of the capital (old households in equilibrium) get paid only a fraction α_j^2 of the value of output, with a fraction $\alpha_j(1-\alpha_j)$ accruing to profits of intermediaries.These profits in turn motivate R&D, described next.

Research and Development An R&D developer that specializes in intermediate good i spends resources of the final consumption good z_{jit} on R&D to achieve innovation. If successful in innovation, the quality of the intermediate good increases from q_{jit-1} to

$$q_{jit} = \lambda q_{jit-1} \tag{13}$$

where the parameter $\lambda > 1$ captures the step size of innovations. The successful innovator immediately becomes the monopolist, and for one period enjoys monopoly profits π_{jit} associated with technology level $q_{jit} = \lambda q_{jit-1}$. In a product line i in which innovation is not successful a randomly chosen entrepreneur becomes the monopolist and produces at quality $q_{jit} = q_{jit-1}$ with associated profits.

We assume that the innovation probability is related to total R&D spending, $(1 + s_{jt})z_{jit}$, –where s_{jt} is a sector-specific R&D government subsidy taken as exogenous by the entrepreneur– relative to the quality after successfully innovating, λq_{jit-1}, given by

$$\phi_j(z_{jit}; q_{jit-1}) = \left(\frac{(1 + s_{jt}) z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma}, \tag{14}$$

with $\gamma \in (0,1)$. The inverse relationship between the success probability and previous quality q_{jit-1} reflects the fact that it becomes increasingly harder to innovate if already a level of quality is reached for variety i. The R&D entrepreneur then spends resources z_{jit} to maximize expected profits according to:

$$\max_{z_{jit}} \left\{ \pi_{jit} \phi_j \left(z_{jit}; q_{jit-1} \right) - z_{jit} \right\}. \tag{15}$$

In an interior solution, optimal R&D spending z_{jit} satisfies the following first-order condition, taking as given the existing quality q_{jit-1}, profits π_{jit} upon success and subsidies s_{jt} by the government.[11]

$$\gamma \left(\frac{(1+s_{jt}) z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma-1} \frac{(1+s_{jt}) \pi_{jit}}{\lambda q_{jit-1}} = 1. \tag{16}$$

The next proposition is a direct consequence of this optimality condition and characterizes the resulting optimal R&D spending and associated innovation probabilities:

Proposition 2. *Total R&D (private and public) spending relative to potential period t technology λq_{jit-1} in each variety i is*

$$\frac{(1+s_{jt}) z_{jit}}{\lambda q_{jit-1}} = \left[\gamma \frac{(1+s_{jt}) \pi_{jit}}{\lambda q_{jit-1}} \right]^{\frac{1}{1-\gamma}} = \left[\gamma (1+s_{jt}) \frac{1-\alpha_j}{\alpha_j} \left(\frac{p_{jt} \alpha_j^2}{R^{\alpha_j}} \right)^{\frac{1}{1-\alpha_j}} l_{jt} \right]^{\frac{1}{1-\gamma}} \tag{17}$$

and the innovation probability in each variety is

$$\phi_j(z_{jit}; q_{jit-1}) = \left(\frac{(1+s_{jt}) z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma} = \left[\gamma (1+s_{jt}) \frac{1-\alpha_j}{\alpha_j} \left(\frac{p_{jt} \alpha_j^2}{R^{\alpha_j}} \right)^{\frac{1}{1-\alpha_j}} l_{jt} \right]^{\frac{\gamma}{1-\gamma}}. \tag{18}$$

Each entity is independent of i. The aggregate share of successful innovations is given by

$$\mu_{jt} = \int \left(\frac{(1+s_{jt}) z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma} di = \left[\gamma (1+s_{jt}) \frac{1-\alpha_j}{\alpha_j} \left(\frac{p_{jt} \alpha_j^2}{R^{\alpha_j}} \right)^{\frac{1}{1-\alpha_j}} l_{jt} \right]^{\frac{\gamma}{1-\gamma}} \tag{19}$$

[11] The Inada condition in the innovation probability guarantees that $z_{jit} > 0$. Since the probability is bounded by 1, there is an endogenous upper bound on z_{jit}. In our quantitative analysis this upper bound is never binding, and in what follows we therefore focus on an interior solution with $\phi_j < 1$.

and is thus independent of the distribution of qualities across varieties i. Aggregate private, public and total resources spent on R&D in sector j are then

$$z_{jt} = \int z_{jit} di = (1+s_{jt})^{\frac{\gamma}{1-\gamma}} \left[\gamma \frac{1-\alpha_j}{\alpha_j} \left(\frac{p_{jt}\alpha_j^2}{R^{\alpha_j}} \right)^{\frac{1}{1-\alpha_j}} l_{jt} \right]^{\frac{1}{1-\gamma}} \lambda \int q_{jit-1} di \quad \text{(20a)}$$

$$= (1+s_{jt})^{\frac{\gamma}{1-\gamma}} \left[\gamma \frac{1-\alpha_j}{\alpha_j} \left(\frac{p_{jt}\alpha_j^2}{R^{\alpha_j}} \right)^{\frac{1}{1-\alpha_j}} l_{jt} \right]^{\frac{1}{1-\gamma}} \lambda q_{jt-1} \quad \text{(20b)}$$

$$z_{jt}^{pub} = s_{jt} z_{jt} \qquad \textit{and} \qquad z_{jt}^{tot} = (1+s_{jt}) z_{jt} \quad \text{(20c)}$$

all of which are independent of the distribution of qualities for each sector j.

Proof. Follows directly from the first-order condition in (16). The second step in (17) uses the profit equation (8) in conjunction with equation (7), as well as the fact that for successfully innovated varieties $q_{jit} = \lambda q_{jit-1}$. □

Given that the aggregate share of successful innovations is given by μ_{jt}, the law of motion for aggregate quality (productivity) in sector j is then

$$q_{jt} = (1-\mu_{jt}) q_{jt-1} + \mu_{jt} \lambda q_{jt-1}. \quad \text{(21)}$$

Quality remains constant when a sector is inactive, $q_{jt} = q_{jt-1}$ as profits are zero and no innovation takes place, $\mu_{jt} = 0$.

3.2 Households

Households derive utility from consumption in young age c_t^y, and old age c_{t+1}^o. Survival from the first to the second period of their life depends on investment i_t into health goods when young which determines the mortality rate households face given by $\pi(i_t)$. We assume that the survival function, $\psi(i) = 1 - \pi(i)$, is increasing in health investment i, $\psi'(i) > 0$ with $\lim_{i \to \infty} \psi(i) = 1$ and $0 \leq \psi(0) < 1$. Finally we assume that the marginal benefit of health investment is finite at $i = 0$, $\psi'(0) < \infty$, and hence households may find it optimal not to spend any resources of longevity-enhancing health goods. The utility of being dead is set to zero; thus, expected lifetime utility is given by

$$(1-\beta) u(c_t^y) + \beta \psi(i_t) u(c_{t+1}^o) \quad \text{(22)}$$

where the period utility function $u(c)$ is at least twice continuously differentiable with $u'(c) > 0$ and $u''(c) < 0$, and satisfies the lower Inada condition, thus $\lim_{c \to 0} u'(c) = \infty$.

Health investment i_t is the composite of two health goods. Health good purchases i_{ct} from the consumption goods sector (basic hygiene and nutritious food), the same sector from which households purchase c_t, and health good purchases i_{ht} of goods from a separate health production sector (modern hospital services and treatments and pharmaceuticals).

$$i_t = f(i_{ct}, i_{ht}). \tag{23}$$

Young households are endowed with one unit of perfectly divisible time which they divide between working in the consumption goods sector, n_{ct}, and the health sector, n_{ht}. Their time constraint therefore reads as

$$n_{ct} + n_{ht} = 1. \tag{24}$$

We envision the representative young household being composed of a large number of members of total measure 1, so that total labor income of the household is given by $w_{ct}n_{ct} + w_{ht}n_{ht}$. Furthermore, households receive transfers T_t implied by accidental bequests from the share of the older generation $1 - \psi(i_{t-1})$ that do not survive until old age. Young households take these transfers as exogenously given. Lastly, government spending is financed by a lump-sum tax τ_t on young households that households also take as exogenous. The utility maximization (22) is subject to the constraints:

$$c_t^y + i_{ct} + p_t i_{ht} + s_t = w_{ct}n_{ct} + w_{ht}n_{ht} + T_t - \tau_t \equiv x_t \tag{25a}$$
$$i_t = f(i_{ct}, i_{ht}) \tag{25b}$$
$$1 = n_{ct} + n_{ht} \tag{25c}$$
$$c_{t+1}^o = Rs_t. \tag{25d}$$

where x_t is cash at hand (CAH) of households after taxes and transfers. Transfers to the generation born in period t due to accidental bequests from generation $t-1$ are given by:

$$T_t = Rs_{t-1}(1 - n_t^o), \tag{26}$$

where $n_t^o = \psi(i_{t-1})$ is the endogenous surviving old population. Transfers are positive if and only if $\psi(i) < 1$, i.e., households die with positive probability after the young age.

3.3 The Government

To capture the role of government spending for the emergence and growth of the modern health sector, we model two government spending programs: a social insurance program which corresponds to Medicare and parts of Medicaid, and a R&D subsidy in the health sector. There is no subsidy in the consumption good sector, $s_{ct} = 0 \; \forall \; t$.

First, we interpret the rising public spending share in modern health spending during the second half of the 20th century (see Figure 2) as being partially driven by the introduction and expansion of public health spending on the old in the form of Medicare and the nursing home component of Medicaid (MC). To account for that public demand, we assume the government covers health spending $p_t i_{ht}^{MC}$ per old capita alive, financed by a lump-sum tax on the current young, τ_t^{MC}. We model public old-age health spending as raising the value of being alive in old-age conditional on survival rather than extending life expectancy.[12] Young households take the expected utility benefit of old-age health spending as given when making their choices, and we calibrate the benefits such that young households are individually indifferent to the public health spending program, that is, such that the expected utility benefit exactly outweighs tax payment.[13]

Second, following Gross and Sampat (2025) among others, we model a R&D subsidy in the health sector in 1940 to capture the role of government spending on research and development in medical and health-related fields during World War II for the emergence and subsequent growth of the modern health sector. As part of the Office of Scientific Research and Development (OSRD), established in 1941, the Committee on Medical Research (CMR) was tasked with overseeing health-related R&D focused on medical research projects such as the development of penicillin, antimalarial drugs, and blood plasma techniques. Therefore, we assume that the government implements an R&D subsidy in the modern health sector in 1940, $s_{h,1940}$, financed by a lump-sum tax τ_{1940}^{RD} on current young workers. Total lump-sum taxes on the young in period t are then given by $\tau_t = \tau_t^{MC} + \tau_t^{RD}$.

[12]We assume public old-age health spending improves the quality rather than the quantity of life following evidence that health changes at older ages often operate through morbidity and functioning rather than additional years of life (Crimmins and Beltrán-Sánchez 2010), and Medicare evidence finding no discernible mortality effects but rather reductions in out-of-pocket spending among the elderly (Finkelstein and McKnight (2008), Engelhardt and Gruber (2011)).

[13]See the calibration Section 5 for a detailed description of the utility function and implementation of the public health spending program.

3.4 Definition of Equilibrium

We define the competitive equilibrium for the aggregate economy, exploiting the aggregation results developed in sections 3.1.

Definition 1. *Given initial conditions* $s_0, q_{c0}, q_{h0}, n_1^o$ *and government policies* $\{s_{ht}, i_{ht}^{MC}\}_{t\geq 1}$, *a competitive equilibrium consists of household allocations* $c_1^o, \{s_t, i_{ct}, i_{ht}, c_t^y, c_{t+1}^o, n_{ct}, n_{ht}\}_{t=1}^{\infty}$, *sectoral aggregates* $\{k_{jt}, y_{jt}, \pi_{jt}, z_{jt}, \mu_{jt}, q_{jt}, l_{jt}\}_{t\geq 1, j\in\{c,h\}}$, *prices* $\{p_t, w_{ct}, w_{ht}\}_{t\geq 1}$, *transfers* $\{T_t\}_{t\geq 1}$ *and taxes* $\{\tau_t^{MC}, \tau_t^{RD}\}_{t\geq 1}$ *such that for all* $t \geq 1$*:*

1. *Given* $(w_{ct}, w_{ht}, p_t, R, T_t, \tau_t)$*, where* $\tau_t = \tau_t^{MC} + \tau_t^{RD}$*, the household allocations* $(s_t, i_{ct}, i_{ht}, c_t^y, c_{t+1}^o, n_{ct}, n_{ht})$ *maximize (22) subject to (25).*

2. *In each active sector* j*, output, profits, and factor prices satisfy*

$$y_{jt} = k_{jt}^{\alpha_j}(q_{jt}l_{jt})^{1-\alpha_j}, \qquad \pi_{jt} = \alpha_j(1-\alpha_j)p_{jt}y_{jt}, \tag{27a}$$

$$w_{jt} = (1-\alpha_j)\frac{p_{jt}y_{jt}}{l_{jt}}, \qquad R = \alpha_j^2\frac{p_{jt}y_{jt}}{k_{jt}}. \tag{27b}$$

   If the health sector is inactive, $l_{ht} = k_{ht} = y_{ht} = \pi_{ht} = 0$*, its wage,* w_{ht}*, and price,* p_{ht}*, are not pinned down.*

3. *In each sector* j*, the innovation intensity* μ_{jt} *and private R&D spending* z_{jt} *satisfy the aggregated optimality conditions (19) and (20a), and sectoral quality evolves according to the law of motion (21).*

4. *The old population evolves according to* $n_t^o = \psi(i_{t-1})$ *and accidental-bequest transfers satisfy* $T_t = Rs_{t-1}(1-n_t^o)$ *for all* $t \geq 2$*.*

5. *The budgets of each government spending program are balanced:*

$$p_t i_{ht}^{MC} n_t^o = \tau_t^{MC}, \qquad s_{ht}z_{ht} = \tau_t^{RD}. \tag{28}$$

6. *The labor markets and the health goods market clear:*

$$l_{jt} = n_{jt}, \; j \in \{c,h\}.$$
$$y_{ht} = i_{ht} + i_{ht}^{MC}n_t^o. \tag{29}$$

Note that in this small open economy the domestic asset market and the final goods market do not have to clear; the net foreign asset position is given by $f_t = s_{t-1} - \sum_j k_{jt}$ and foreigners receive Rf_t final goods from the domestic economy.

3.5 Discussion of Main Model Assumptions and Extensions

In this section we discuss and justify the main simplifying assumptions we have made to characterize the dynamic equilibrium. First, households in our model privately spend on health goods and services only when young, and these expenditures only impact the chance of survival to the second period of life, and thus life expectancy. Therefore we abstract from the potentially direct positive impact of health investments on labor productivity and associated income at the individual level. However, larger health spending encourages R&D and thus aggregate productivity (and therefore income) growth. Finally, observe that even though individuals do not purchase health goods privately when old, the government does so on their behalf. These public health investments arguably crowd out private medical expenditures (see e.g., Cutler and Gruber (1996)), and we implicitly make the (admittedly stark) assumption that this crowding-out is complete.

Second, public spending on health (through Medicare, and to some degree, Medicaid), occurs only when households are old. This model assumption does ignore certain important government programs that generate demand for health goods on behalf of the young (most notably, the part of Medicaid not related to nursing home expenditures for the poor old). Note, however, that unless the size of the public program is already too large, private households will supplement it by private health expenditures when young (which we do model), and if public and private health spending are perfect substitutes and the former is financed by non-distortionary taxes, the private health expenditure optimality conditions will not be affected by the presence of public health spending when the household is young.

Third, we abstract from household heterogeneity within generations since our focus is on aggregate health spending and life expectancy, and in our model demand from the entire population contributes to the incentives to innovate, and the associated technological progress. This simplification allows for a model that is analytically tractable, at the expense of abstracting from heterogeneity in life expectancy and the fact that, in practice, demand for new frontier technologies by affluent households may have driven an important part of medical innovations.[14] These assumptions are commonly used in important papers in the literature that our work builds on, notably Hall and Jones (2007) and Jones (2016).

4 Theoretical Characterization of the Health Transition

The health transition of the model is characterized by the following phases:

1. In Phase 1, income (CAH x_t) is too low to make it optimal for households to invest any resources in health that would lead to an increase in life expectancy: for all $t < T_1$ we have $i_{ct} = i_{ht} = 0$ and $\psi(i_t) = \psi_0$ and $c_t^y + c_{t+1}^o/R = x_t$.

2. In Phase 2, CAH has risen sufficiently that households start to spend on basic health goods (e.g., better hygiene and nutrition), but not yet on modern medical services produced by a high-tech medical sector: $i_{ct} > 0$ and $i_{ht} = 0$ and $\psi(i_t) > \psi_0$, for all $t \in [T_1, T_2)$. Life expectancy now rises with income over time, $\psi(i_{t+1}) > \psi(i_t)$.

3. In Phase 3, a further increase in x_t induces spending on modern health goods and life expectancy rises further: For all $t \geq T_2$ we have $i_{ct} > 0$ and $i_{ht} > 0$ and life expectancy grows further $\psi(i_{t+1}) > \psi(i_t)$.

4. Phase 3 converges to an asymptotic balanced growth path (BGP) with constant expenditure shares on modern and basic health goods, consumption goods, and a constant relative price of health goods p_t: for $t \to \infty$, we have $p_t \to p^*$, $\vartheta_{t,c} = \frac{c_t^y}{x_t} \to \vartheta_c^* \in [0,1]$, $\vartheta_{t,e} = \frac{e_t}{e_t+s_t} = \frac{p_t i_{ht} + i_{ct}}{(1-\vartheta_{t,c})x_t} \to \vartheta_e^* \in [0,1]$ and $\vartheta_{t,ic} = \frac{i_{ct}}{e_t} \to \vartheta_{ic}^*$.

To establish the emergence of these phases and the BGP, we will make specific functional form assumptions that we maintain in the quantitative analysis.

4.1 Characterization of the Household Problem in Partial Equilibrium

Given the relative price of health goods p_t, wages (w_{ct}, w_{ht}) and transfers net of taxes $T_t - \tau_t$ which determine CAH x_t, young households choose the labor allocation n_{ct}, n_{ht}, the health investment allocation i_{ct}, i_{ht} (and the implied total health investment i_t, survival probability $\psi(i_t)$ and health expenditures $e_t = i_{ct} + p_t i_{ht}$), as well as consumption in both periods of their life c_t^y, c_{t+1}^o, to maximize (22) subject to (25).

[14] An extension to within-cohort household heterogeneity by income would be important to explain the recent decline in life expectancy for specific sub-populations in the U.S., especially white men without a college degree, see, e.g., Case and Deaton (2020). From the perspective of our model, the relative lack of income growth for this population would lead to a reduction in the investment into longevity among this population, and the associated early deaths that are a main theme of Case and Deaton (2020).

Labor Supply Allocation Since labor supply is perfectly substitutable across the two production sectors, an interior allocation with both sectors active requires wages to be equal across sectors, otherwise a corner solution for labor supply emerges: for $j, i \in \{c, h\}$,

$$n_{jt} = \begin{cases} 1 & \text{if } \; w_{jt} > w_{it} \\ \in [0,1] & \text{if } \; w_{jt} = w_{it} \\ 0 & \text{if } \; w_{jt} < w_{it}. \end{cases}$$

The Division of Health Expenditures For given expenditure e_t on health and relative price of modern health goods p_t, we now characterize the optimal allocation of e_t across basic and modern health goods, (i_{ct}, i_{ht}). The solution to this problem determines health investment $i(e_t, p_t)$ and the survival probability $\psi(i_t)$ as a function of (e_t, p_t). To do so, we assume that (i_{ct}, i_{ht}) are aggregated into effective health investment i_t according to the following non-homothetic CES specification

$$i_t = \left[\chi \left(i_{ht} + \nu\right)^{\rho} + (1 - \chi) \left(i_{ct} + \nu\right)^{\rho}\right]^{\frac{1}{\rho}} \tag{30}$$

Here $\chi \in (0, 1)$ is the share parameter, $\rho \leq 1$ governs the elasticity of substitution between basic and modern health goods, given by $\varepsilon = \frac{1}{1-\rho}$, and $\nu > 0$ allows for a non-homotheticity. Note that without any health investments $(i_c = i_h = 0)$, the implied $i = \nu > 0$ and thus $\psi_0 \equiv \psi(\nu) = 1 - (1 + \nu)^{-\xi}$ gives the (positive) probability of survival in the absence of any health investment. The household maximizes objective (30) subject to the static budget constraint

$$p_t i_{ht} + i_{ct} = e_t. \tag{31}$$

In the above specification of health investment, the non-homotheticity $\nu > 0$ bounds marginal effective health investment with respect to both health goods and thus total health spending from above

$$\left.\frac{\partial i_t}{\partial i_{jt}}\right|_{i_{jt}=0} < \infty \quad j \in c, h \quad \Rightarrow \quad \left.\frac{\partial i_t}{\partial e_t}\right|_{e_t=0} < \infty, \tag{32}$$

which allows for the possibility of corner solutions in health spending. Depending on (e_t, p_t), the allocation is then interior $(i_{ct}, i_{ht}) > 0$ or involves corner solutions for either $i_{ht} = 0$ or for $i_{ct} = 0$. The following proposition summarizes the solution:

Proposition 3. *For health expenditure $e_t \geq 0$ and price of health goods $p_t > 0$, define*

$$\underline{p} = \frac{\chi}{1-\chi} \tag{33a}$$

$$\underline{e}^c(p_t) = \nu p_t \left(\left(\frac{\underline{p}}{p_t} \right)^{\varepsilon} - 1 \right) \tag{33b}$$

$$\underline{e}^h(p_t) = \nu \left[\left(\frac{p_t}{\underline{p}} \right)^{\varepsilon} - 1 \right] \tag{33c}$$

and note that $\underline{e}^c(p_t) > 0$ if (and only if) $p_t < \underline{p}$, and $\underline{e}^h(p_t) > 0$ if (and only if) $p_t > \underline{p}$. Then the optimal allocation of health goods is given as follows:

(1) Suppose $e_t = 0$. Then, trivially, $i_{ct} = i_{ht} = 0$ and $i(0, p_t) = \nu$.

(2) Suppose $p_t \geq \underline{p}$ and $e_t \in (0, \underline{e}^h(p_t)]$. Then $i_{ht} = 0$ and $i_{ct} = e_t$.

(2b) Suppose $p_t \leq \underline{p}$ and $e_t \in (0, \underline{e}^c(p_t)]$. Then $i_{ct} = 0$ and $i_{ht} = e_t/p_t$.

(3) Suppose $p_t \geq \underline{p}$ and $e_t > \underline{e}^h(p_t)$ or $p_t \leq \underline{p}$ and $e_t > \underline{e}^c(p_t)$. Then $(i_{ct}, i_{ht}) > 0$ and

$$i_{ct} = \tilde{\vartheta}(p_t)\tilde{e}_t - \nu = \tilde{\vartheta}(p_t)e_t + \nu \left(\tilde{\vartheta}(p_t)(1+p_t) - 1 \right) \tag{34a}$$

$$p_t i_{ht} = (1 - \tilde{\vartheta}(p_t))\tilde{e}_t - p_t\nu = (1 - \tilde{\vartheta}(p_t))e_t + \nu \left((1 - \tilde{\vartheta}(p_t))(1+p_t) - p_t \right) \tag{34b}$$

$$i(e_t, p_t) = \frac{\tilde{e}_t}{P(p_t)} = \frac{e_t + (1+p_t)\nu}{P(p_t)}, \tag{34c}$$

where $\tilde{e}_t \equiv e_t + (1+p_t)\nu$ denotes homothetic health expenditures,

$$\tilde{\vartheta}(p_t) = \left[1 + \left(\frac{\chi}{1-\chi} \right)^{\varepsilon} p_t^{1-\varepsilon} \right]^{-1} = (1-\chi)^{\varepsilon} P(p_t)^{\varepsilon-1} \in (0,1) \tag{35}$$

is the homothetic health spending share on basic health, and

$$P(p_t) = \left[(1-\chi)^{\varepsilon} + \chi^{\varepsilon} p_t^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{36}$$

is the unit-cost (CES) price index of effective health investment $i(e_t, p_t)$.[15]

Proof. See Appendix B.2.1. □

The key parts of this proposition are items (2) and (3) which state that if the price of modern health goods is too high relative to the desired health spending, then households

[15] In our quantitative calibration, we focus on $\rho > 0$ throughout the paper.

find it optimal to only spend on basic health goods. Conversely, if p_t is sufficiently low and e_t high, then households spread health spending across both sectors.[16] The possible existence of both corners is driven by the presence of the non-homotheticity, $\nu > 0$.

The potential presence of corner solutions in Proposition 3 gives rise to the following potential dynamics of health spending, which emerge in our quantitative implementation. Initially households are poor and find it optimal to not spend anything on health ($e_t = 0$). As incomes increase but the price of modern health good remains relatively high, $p_t > \underline{\mathrm{p}}$, households optimally set $e_t \in (0, \underline{\mathrm{e}}^h(p_t)]$, and the economy enters phase 2 where the modern health sector continues to be inactive. Finally, further income growth increases e_t and public R&D subsidies in the modern health sector during World War II reduce p_t so that the conditions of item (3) in the proposition are satisfied, and the economy enters phase 3 in which households purchase health goods from both sectors. Of course, this discussion presupposes a specific path for overall health expenditures $\{e_t\}$; we next characterize this path as well as the required conditions for the optimality of $e_t = 0$, jointly with the savings decision in financial assets (and the implied consumption in both periods), for a given sequence of relative health prices $\{p_t\}$.

The Level of Health Expenditures Given the optimal division of health expenditures e_t and resulting health investment $i(e_t, p_t)$, which enters the survival function $\psi(i) = \psi(i(e_t, p_t))$, we now consider the problem of allocating CAH x_t between consumption c_t^y when young, health expenditures e_t and consumption when old, $c_{t+1}^o = R(x_t - c_t^y - e_t)$ implied by these choices. Given x_t and p_t, households solve

$$v(x_t, p_t) = \max_{0 \leq c_t^y, e_t \leq x_t} \left\{(1-\beta)u(c_t^y) + \beta\psi\left(i(e_t, p_t)\right) u(R(x_t - c_t^y - e_t))\right\}. \tag{37}$$

We denote the solutions as $c^y(x_t, p_t)$ and $e(x_t, p_t)$. Because of the Inada conditions on $u(.)$, we can ignore potential corner solutions for c_t^y. The first order conditions with respect to (c_t^y, e_t) are

$$(1-\beta)u'(c_t^y) = \beta R\psi\left(i(e_t, p_t)\right) u'(R(x_t - c_t^y - e_t)) \tag{38a}$$

$$\psi'\left(i(e_t, p_t)\right) \frac{\partial i(e_t, p_t)}{\partial e_t} u(R(x_t - c_t^y - e_t)) \leq R\psi\left(i(e_t, p_t)\right) u'(R(x_t - c_t^y - e_t)), \tag{38b}$$

where the second equation holds with equality if $e_t > 0$. The first equation is the standard intertemporal Euler equation for consumption. The second equation trades off

[16] Part (2b) of the proposition delineates the possibility that all health spending is allocated to the modern sector, but this does not occur in the transitions considered in the quantitative part of the paper.

the benefits of spending an extra unit of resources on health goods today on the left hand side with the utility loss from one unit of foregone consumption tomorrow on the right hand side. These costs and benefits are equalized whenever health spending is strictly positive; if the costs exceed these benefits at $e_t = 0$, it is optimal to not spend on health at all. We now establish the conditions for the existence of a corner with $e_t = 0$. To do so, we assume that the probability of survival, as a function of health investment, follows a type 2 Pareto distribution, given by

$$\psi(i) = 1 - (1+i)^{-\xi}. \tag{39}$$

with $\xi > 0$.[17] Note that $\psi(\cdot)$ is strictly increasing and strictly concave in i, with $0 < \psi'(i) \leq \xi$ and $\psi''(i) < 0$ for all $i \geq 0$ and $\lim_{i\to\infty}\psi(i) = 1$. Combining this with equation (32), we observe that the marginal survival benefit from health spending, $\frac{d}{de}\psi(i(e,p))$, is bounded from above at the corner $e_t = 0$ since both $\psi'\left(i(0,p_t)\right)$ and $\frac{\partial i(0,p_t)}{\partial e_t}$ are bounded from above. Therefore, households may find it optimal not to invest in health $e_t = i_{ct} = i_{ht} = 0$ as the following proposition establishes.

Proposition 4. *Fix a relative health price $p > 0$. Then given the survival specification $\psi(i)$ and the assumptions on period utility, there exists a threshold CAH level $\underline{x}_1(p) > 0$ such that for all $x \leq \underline{x}_1(p)$, the household's optimal health spending is zero, $e(x,p) = 0$.*

Proof. See Appendix B.2.2. □

Proposition 4 characterizes the extensive margin of total health spending. Given our functional form assumptions, it states that households optimally choose not to spend on health as long as their CAH is sufficiently low for any positive relative health price. Moreover, the threshold $\underline{x}_1(p)$ is independent of the price and only depends on parameters if households allocate their first unit of health spending to basic rather than modern health goods as characterized in Proposition 3.

Lemma 1. *Fix a relative health price $p > \underline{p}$. Then condition (38b), evaluated at the extensive margin $e = 0$, is independent of p. Hence, the CAH threshold characterized*

[17] The general form of our mortality function can be seen as $\pi(i) = 1 - \psi(i) = (\kappa + i)^{-\xi}$. For $\kappa = 0$ it captures the specification of Jones (2016) which exhibits a constant elasticity of the mortality risk with respect to health investment equal to ξ and to which our specification converges to in the long run, as $i \to \infty$. Our specification has the advantage that the mortality (and thus survival) rate is always contained in $[0, 1]$. Moreover, note that even for $\kappa = 0$ a corner solution for health spending exists as $\psi'(i_0 = \nu) < \infty$ still holds.

in Proposition 4 is also independent of p on this region, and we denote the common threshold as $\underline{x}_1(p) = \underline{x}_1(\underline{p})$ for all $p > \underline{p}$. Moreover, for any CAH $x \geq 0$, positive private demand for modern health goods requires

$$p < \bar{p}(x) \equiv \underline{p}\left(1 + \frac{x}{\nu}\right)^{1/\epsilon}. \tag{40}$$

In particular, $\bar{p}(x) > \underline{p}$ for all $x > 0$.

Proof. See Appendix B.2.2. □

Our calibration strategy exploits this separation in the household problem implied by Lemma 1. Given CAH x_t and $p_t > \underline{p}$, households first spend on basic before modern health goods, and the timing of the first kickoff in health spending from Phase 1 to Phase 2 is governed by $\underline{x}_1(\underline{p})$. Thus the observed kickoff in life expectancy disciplines $\underline{p} = \chi/(1-\chi)$. Conditional on $x_t > \underline{x}_1(\underline{p})$, the second kickoff in health spending from Phase 2 to Phase 3, when households first adopt modern health goods, is then governed by the relative price of health goods p_t and requires it to fall below the household-side upper bound $\bar{p}(x_t)$. We next partially characterize the dynamic equilibrium which determines the evolution of (p_t, x_t) and show that the observed second kickoff – the timing of the emergence of modern health – disciplines the initial productivity in the modern health sector q_{h0}, and thereby also ensures that any price consistent with an interior equilibrium satisfies $p_t > \bar{p}(x_t) > \underline{p}$ during Phases 1 and 2, ruling out positive modern health production.

4.2 Characterization of the Equilibrium

In any period t equilibrium with an active health sector, which we refer to as an interior (period t) equilibrium, labor across sectors $l_t = (l_{ht}, l_{ct})$ and the relative price of health goods p_t are jointly determined to clear the labor market (24) and health goods market (29), with $l_{ht} > 0$. Given the state of the economy, and specifically, the productivity in both sectors (q_{ct-1}, q_{ht-1}), it may be the case that the maximum health price at which households are willing to purchase modern health goods in positive amounts (characterized by Proposition 3) is so low that firms, given those productivities, cannot cover the cost of production. In that case the equilibrium labor allocation is given by $(l_{ht}, l_{ct}) = (0, 1)$, and the health price is not determined.

To determine a lower bound on the health price for an interior equilibrium to exist from the production side, recall that labor supply is positive in both sectors only if wages are equalized across the two sectors. Exploiting the solution (7) to the intermediate good producer's maximization problem, we can rewrite the first-order condition (3) for labor demand from final good producers in each sector as

$$w_{jt} = p_{jt}^{\frac{1}{1-\alpha_j}} (1-\alpha_j) \left(\frac{\alpha_j^2}{R}\right)^{\frac{\alpha_j}{1-\alpha_j}} q_{jt}. \tag{41}$$

Since consumption is the numeraire, $p_{ct} = 1$, wage equalization across sectors requires

$$1 = \frac{w_{ht}}{w_{ct}} = p_t^{\frac{1}{1-\alpha_h}} C(\alpha_h, \alpha_c, R) \frac{q_{ht}}{q_{ct}} = p_t^{\frac{1}{1-\alpha_h}} C(\alpha_h, \alpha_c, R) r_{qt}, \tag{42}$$

where $r_{qt} = q_{ht}/q_{ct}$ is the ratio of qualities (productivities) between both sectors and $C(\alpha_h, \alpha_c, R)$ is a constant.[18] Thus, the requirement that wages must be equal in an interior equilibrium gives rise to a relationship between the health price p_t and relative productivity r_{qt} that must hold in every period in which both sectors are active. Recall that productivities today (q_{ct}, q_{ht}) are determined by their predetermined values (q_{ct-1}, q_{ht-1}) in $t-1$ as well as the share of varieties (μ_{ct}, μ_{ht}) that innovate, see (21). Using (21) in (42) and rewriting yields a condition that must hold in an interior equilibrium:

$$p_t = \left(\frac{1}{C(\alpha_h, \alpha_c, R)} \frac{[1 + (\lambda - 1)\mu_{ct}(1, l_{ct})]\, q_{ct-1}}{[1 + (\lambda - 1)\mu_{ht}(p_t, l_{ht})]\, q_{ht-1}}\right)^{1-\alpha_h}. \tag{43}$$

Since both innovation probabilities μ_{jt} are bounded between 0 and 1, equation (43) imposes a lower bound on the health price below which labor markets cannot clear, and thus constitutes a necessary entry condition for the health sector to be active given by:

$$p_t \geq \left(\frac{1}{C(\alpha_h, \alpha_c, R)} \frac{q_{ct-1}}{\lambda q_{ht-1}}\right)^{1-\alpha_h} \equiv \underline{\mathrm{p}}_t(r_{qt-1}), \tag{44}$$

which depends on the predetermined productivity ratio $r_{qt-1} = \frac{q_{ht-1}}{q_{ct-1}}$. Intuitively, if $p_t < \underline{\mathrm{p}}_t(r_{qt-1})$, then the health sector is too unproductive relative to the consumption sector, even when all varieties in the former innovate and none in the latter, to be able to profitably produce while matching consumption-sector wages given the price p_t of its output. Consequently, if $\underline{\mathrm{p}}_t(r_{qt-1})$ exceeds the highest relative price consistent with positive modern health demand from households, $\bar{p}(x_t)$, then the health sector must necessarily be inactive.

[18] This constant is explicitly given by $C(\alpha_h, \alpha_c, R) \equiv \frac{1-\alpha_h}{1-\alpha_c} \left(\frac{\alpha_h^2}{R}\right)^{\frac{\alpha_h}{1-\alpha_h}} \left(\frac{\alpha_c^2}{R}\right)^{-\frac{\alpha_c}{1-\alpha_c}}$.

Corollary 1. *Assume* $i_{ht}^{MC} = 0$. *If*

$$\underline{p}_t(r_{qt-1}) \geq \bar{p}(x_t) = \underline{p}\left(1 + \frac{x_t}{\nu}\right)^{1/\epsilon},$$

then $y_{ht} = i_{ht} = 0$. *Since* $\underline{p}_t(r_{qt-1})$ *is strictly decreasing in* $r_{q,t-1}$, *this scenario emerges if the health sector is sufficiently unproductive relative to the consumption sector at the beginning of period t.*

Proof. Immediate from Lemma 1 and the production-side entry condition (44). □

4.3 Characterization of the Balanced Growth Path

Formally, we consider an asymptotic interior balanced growth path (BGP) as $t \to \infty$ and $x_t \to \infty$ on which (i) the relative price of modern health goods is constant and finite (and denoted by $p^* \in (0, \infty)$), and (ii) households spend an interior share of their CAH on health goods $\frac{e_t}{x_t} \in (0, 1)$.

"Demand": Household Choices on the BGP In addition to the survival function being of form (39), we now assume that the period utility function is of the form:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + b^a, \qquad a \in \{y, o\} \tag{45}$$

where $\sigma > 0$ is the curvature parameter controlling the intertemporal elasticity of substitution, and the parameter $b^a > 0$ measures the pure value of being alive at age a. Without loss of generality, we normalize the young-age value to zero, $b^y = 0$, and denote the value of surviving to old age as $b^o = b$, which will ensure that flow utility is positive and survival is desirable.[19] Using the utility function and given interior health expenditures as share of CAH $\frac{e_t}{x_t} \in (0, 1)$, the optimality condition for health expenditures in (38b) can be written as[20]

$$c_{t+1}^o \left(b(c_{t+1}^o)^{\sigma-1} + \frac{1}{1-\sigma} \right) = \frac{P(p_t)R}{\xi}(1+i_t)\left((1+i_t)^{\xi} - 1\right) \tag{46}$$

which gives rise to the following Proposition:

Proposition 5. *Given a constant health price* p^*, *household health spending in CAH is interior along an asymptotic BGP,* $e_t/x_t \in (0, 1)$, *if and only if* $\xi = \sigma - 1$. *Given*

[19] The functional form assumption follows Hall and Jones (2007).

[20] Using the budget constraint for c_{t+1}^o and the fact that $\frac{\partial i(e_t, p_t)}{\partial e_t} = P(p_t)^{-1}$ from equation (34c).

$\xi = \sigma - 1$, any such asymptotic BGP is characterized by household allocations that converge to interior constant shares in CAH. It is characterized by the two spending shares $\vartheta_c^* = \frac{c_t^y}{x_t} \in (0,1)$ (the share of young consumption in CAH) and $\vartheta_e^* = \frac{e_t}{e_t+s_t} \in (0,1)$ (the health expenditure share in total "saving" $e_t + s_t$) along the BGP given by:

$$\vartheta_e^* = \left(1 + \left[\frac{(P(p^*)R)^{1-\sigma}}{b\xi}\right]^{\frac{1}{\sigma}}\right)^{-1} \in (0,1) \tag{47a}$$

$$\vartheta_c^* = \left(1 + \frac{\left(\frac{\beta}{1-\beta}R^{1-\sigma}\right)^{\frac{1}{\sigma}}}{1-\vartheta_e^*}\right)^{-1} \in (0,1). \tag{47b}$$

All other expenditure shares (out of CAH) follow and on the asymptotic BGP we have $\psi(i) \to \psi^* = 1$.[21]

Proof. See Appendix B.3.1. □

Since $\xi > 0$, the condition $\xi = \sigma - 1$ implies $\sigma > 1$ which we assume throughout. Demand for modern health goods comes from households and the government's public health system, that is, $p_t i_{ht} + \tau_t^{MC}$. Using the government budget constraint (28), demand for modern health goods as fraction of GDP along the BGP can be expressed as

$$D(p^*) \equiv \lim_{t\to\infty} \frac{p_t i_{ht} + \tau_t^{MC}}{y_t} = \vartheta(p^*)\left[(1-\alpha_c) + (\alpha_c - \alpha_h)S(p^*) - \hat{\tau}^{*MC}\right] + \hat{\tau}^{*MC} \tag{48}$$

where $\hat{\tau}^{*MC}$ is the long-run size of public health spending as a share of GDP, $\vartheta(p^*) = (1-\tilde{\vartheta}(p^*))(1-\vartheta_c^*(p^*))\vartheta_e^*(p^*) \in (0,1)$ is the share of household CAH spent on modern health goods and $\left[(1-\alpha_c) + (\alpha_c - \alpha_h)S(p^*) - \hat{\tau}^{*MC}\right]$ is the share of CAH in GDP along the BGP which depends on the health output share $S(p^*) \equiv \frac{p^* y_{ht}}{y_t}$.[22]

"Supply": Allocation of Labor and Output along the BGP Combining the labor market clearing condition (42) with the fact that growth rates of qualities are equalized along the BGP yields the following characterization:

[21] Specifically, along the BGP, we have $\frac{e_t}{x_t} = \vartheta_e^*(1-\vartheta_c^*)$, $\frac{s_t}{x_t} = (1-\vartheta_e^*)(1-\vartheta_c^*)$, $\frac{c_{t+1}^o}{x_t} = R(1-\vartheta_e^*)(1-\vartheta_c^*)$, with health investments along the BGP given by $i_{ct} = i_c(e_t, p^*) = i_c(\vartheta_e^*(1-\vartheta_c^*)x_t, p^*), p_t i_{ht} = p^* i_h(e_t, p^*) = p^* i_h(\vartheta_e^*(1-\vartheta_c^*)x_t, p^*)$.

[22] See Appendix B.3.3 for the derivation.

Proposition 6. *In an interior asymptotic BGP with modern health price p^*, the labor market clearing condition pins down the constant quality ratio $r_q^* \equiv \frac{q_{ht}}{q_{ct}}$ given by:*

$$r_q^* = C(\alpha_h, \alpha_c, R)^{-1} (p^*)^{-\frac{1}{1-\alpha_h}} . \tag{49}$$

Qualities in both sectors grow at the same rate due to constant and equal innovation probabilities $\mu_{ct} = \mu_{ht} = \mu^*$; the constant labor and output allocations are given by:

$$r_l^* \equiv \frac{l_h^*}{l_c^*} = \frac{\alpha_c}{\alpha_h} r_q^*, \tag{50}$$

$$r_y^* \equiv \frac{p^* y_{ht}}{y_{ct}} = \frac{1-\alpha_c}{1-\alpha_h} \frac{\alpha_c}{\alpha_h} r_q^*. \tag{51}$$

As a result, qualities, (q_{ht}, q_{ct}), sectoral and aggregate production and intermediate inputs, $(y_{ht}, y_{ct}, y_t, k_{ht}, k_{ct}, k_t)$, grow at the constant quality growth rate g_q^* given by:

$$g_q^* = 1 + \mu^*(\lambda - 1). \tag{52}$$

The constant supply-side health output share as function of the relative modern health price, denoted by $S(p^*) \equiv \frac{p^* y_{ht}}{y_t}$, is given by:

$$S(p^*) \equiv \frac{p^* y_{ht}}{y_t} = \frac{r_y^*}{1 + r_y^*} = \frac{(1-\alpha_c)\alpha_c}{(1-\alpha_c)\alpha_c + (1-\alpha_h)\alpha_h C(\alpha_h, \alpha_c, R)\,(p^*)^{\frac{1}{1-\alpha_h}}}. \tag{53}$$

Proof. See Appendix B.3.2. □

The proposition shows that the relative labor allocation is proportional to the relative productivity across the two sectors. Together with $l_c^* + l_h^* = 1$ (which has to hold in any equilibrium), equation (50) determines the allocation of labor on the BGP, conditional on the long-run equilibrium health price p^*. Relative output and output shares in the proposition then immediately follow. It remains to determine the health price on the BGP from the market clearing condition in the health goods sector.

Equilibrium in the Market for Health Goods on the BGP Equating the production health share (53) to the demand health share (48) in the modern health sector determines the relative price p^* of health goods along the asymptotic BGP:

$$S(p^*) = D(p^*) \tag{54}$$

The next proposition states that such a BGP relative health price satisfying (54) exists, and states conditions under which it is unique.

Proposition 7. *Assume interior long-run public health spending $\hat{\tau}^{MC} \in (0,1)$ and maintain $\xi = \sigma - 1 > 0$. Then an interior asymptotic BGP with health price $p^* > 0$ satisfying equation (54) exists. Furthermore, let*

$$\eta_e(p) \equiv \frac{d \log (e_t/x_t)}{d \log P(p)}, \qquad \eta_{i_h}(p) \equiv \frac{d \log (p_t i_{ht}/e_t)}{d \log P(p)}, \tag{55}$$

denote the elasticity of the health expenditure share in CAH and the elasticity of the modern health spending share in total health spending with respect to the health price index $P(p)$, respectively. Then the following two conditions are sufficient (but not necessary) for the interior BGP health price $p^* > 0$ and, thus, the BGP to be unique:

$$\hat{\tau}^{MC} < 1 - \max\{\alpha_c, \alpha_h\}, \tag{56}$$

$$0 \leq \eta_e(p_\tau) + \eta_{i_h}(p_\tau), \tag{57}$$

where $p_\tau > 0$ is the unique solution to $S(p_\tau) = \hat{\tau}^{MC}$.

Proof. See Appendix B.3.4. □

Existence follows fairly straightforwardly from the fact that both $S(p), D(p)$ are continuous functions with range in $[0,1]$ and that $S(p)$ is strictly decreasing with $\lim_{p \downarrow 0} S(p) = 1$ and $\lim_{p \to \infty} S(p) = 0$. Uniqueness is more difficult to establish because the function $D(p)$ can be non-monotonic in the price if the two health goods are relatively substitutable. To see this, consider a fall in the modern health price p and thus the price of effective health $P(p)$. If the two health goods are substitutable, the share of health spending in the modern sector, pi_{ht}/e_t, rises, so the within-health elasticity $\eta_{i_h}(p)$ is negative. At the same time, the health spending share in CAH, e_t/x_t, falls because consumption and overall health are complements (as old-age consumption only delivers utility conditional on being alive), so the overall elasticity $\eta_e(p)$ is positive. The conditions in Proposition 7 ensure that the positive scale effect dominates the negative within-health substitution effect on the relevant price range. As a result, the excess-demand function $E(p) \equiv D(p) - S(p)$ is increasing, which delivers uniqueness of the interior BGP price. We verify in the quantitative analysis that these sufficient conditions hold.

5 Calibration

We interpret a model period as 40 years. The first period covers age 20-60, and the second period is accordingly 60-100. Remaining cohort life expectancy at age 20 in the model is then $(1 + \psi(i)) \cdot 40$ years. We set a subset of parameters externally to standard values from the literature and we choose the parameters governing the government spending programs to match their size. We report those in Table 2. We then internally calibrate the remaining parameters and choose the initial conditions to match a set of empirical moments, summarized in Table 3. We discuss the calibration strategy for the key parameters below. While they are jointly determined, we associate each with the moment that is most informative for its identification.[23]

Phases and calendar years. Table 1 summarizes the stages of the health transition and the corresponding years. Recall from Figure 1 that remaining cohort life expectancy at age 20 first starts to rise sometime between 1820 and 1860. Thus, we interpret the model years 1780 and 1820 as phase 1, and 1860 as the year by which the rise in per capita income has been sufficient for individuals to spend on basic health goods such as hygiene and nutrition. We then treat the year 1940 as the onset of modern medical times, given our interpretation of Figure 2. Accordingly, the years 1860 and 1900 correspond to phase 2 and the years from 1940 onwards correspond to phase 3.

Table 1: Phases and Calendar Years

1780	1820	1860	1900	1940	1980	2020	$t \to \infty$
Phase 1		Phase 2		Phase 3			BGP

Health investment. We calibrate (ξ, ν, χ, ρ), which govern the health investment (30) and survival function (39) and thus the mapping from households' health spending choices to life expectancy, to match the empirical properties of the health transition. We calibrate ξ targeting an (absolute) elasticity of the mortality rate with respect to modern health spending for a 60-year-old from Hall and Jones (2007) in 1980, which they estimate to be 0.16 between 1950 and 2000. We calibrate the non-homotheticity parameter ν such that life expectancy in phase 1, pinned down by initial survival $\psi_0 = 1 - (1+\nu)^{-\xi}$, matches $\text{LE}_{1820} = 40.44$ years. We calibrate the relative shares of modern and basic health in the health investment function, χ, to match life expectancy at the first kickoff as outlined in

[23] All details of our approach are described in Appendix C.

Section 4.1.[24] Lastly, we discipline ρ, which governs the substitution elasticity between basic and modern health goods, to match the rise in modern spending share as a share of GDP between 1940 and 1980.[25]

Table 2: External Calibration

	Parameter		Moment/Source	Data	Model
Small open economy					
R	Gross real return factor	3.3	Annualized net real rate	standard	3%
Households					
β	Weight on old-age utility	0.16	Annual discount factor	0.96	0.96
Production and R&D					
α_c	capital share consumption	0.33	Capital share U.S.	0.33	0.33
α_h	capital share health	0.22	U.S. health sector capital share Acemoglu and Guerrieri (2008)	0.22	0.22
γ	Innovation curvature	0.35	Patent elasticity w.r.t R&D Hausman et al. (1984)	0.35	0.35
Government					
τ_{1980}^{MC}	1980 public health tax	28.42	Old-age public health in GDP	1.57%	1.57%
τ_{2020}^{MC}	2020 public health tax	151.66	Old-age public health in GDP	4.32%	4.32%
$s_{h,1940}$	Health R&D subsidy	0.2	CMR program	0.2	0.2

Preferences. We discipline the base value-of-life, b, by targeting a value of a statistical life (VSL) of \$3 million in 2000, following Hall and Jones (2007), who use a U.S. Department of Transportation benchmark.[26] We compute the VSL as the willingness to pay by the young for a marginal increase in survival to old age as is standard in the literature.[27]

Government programs. We set the size of public health spending on the old to zero prior to the introduction of the U.S. Medicare system in 1965, $\hat{\tau}_{t<1980}^{MC} = 0$. In 1980 and 2020, we calibrate the tax to match Medicare spending plus the nursing and care home

[24] In particular, we calibrate $\underline{p} = \chi/(1-\chi)$ such that the CAH threshold satisfies $x_{1820} < \underline{x}_1(\underline{p}) < x_{1860}$ to match the timing of the first kickoff and, within these bounds, we target life expectancy in 1860.

[25] In Appendix C.7, we show that the evolution of the modern health spending share over time is highly sensitive to ρ, and thus informative to discipline ρ. Similarly, we show that the *cross-sectional* income-life expectancy gradient is not very informative about ρ. The key distinction between the cross-section and the time series is that in the latter not only income but also the price of modern health goods is changing.

[26] Empirical estimates of the value of a statistical life are dispersed, ranging from \$2 million to \$9 million dollars as discussed by Viscusi and Aldy (2003).

[27] The willingness to pay for a marginal increase in survival for generation t is given by

$$VSL_t = \frac{\beta}{1-\beta} \frac{u(c_{t+1}^o)}{u'(c_t^y)}. \tag{58}$$

We construct the year-2000 model value as the average of 1980 and 2020. To express the VSL in constant 2000 dollars, we multiply VSL_t/y_t by real U.S. GDP per capita in 2000 dollars.

component of Medicaid.[28] From 2020 onward, we assume that per-old-capita spending in GDP remains constant, $\hat{\tau}_t^{MC}/n_t^o = \hat{\tau}_{2020}^{MC}/n_{2020}^o \; \forall \; t \geq 2020$. The size of the program then converges to $\hat{\tau}^{*MC} = \hat{\tau}_{2020}^{MC}/n_{2020}^o$ on the BGP. To capture the corresponding utility benefit, we let public health spending raise the value of being alive in old-age by b_t^{MC}, where the cohort-specific value b_t^{MC} is such that the period-t young household is individually indifferent to participating, and paying the tax τ_t^{MC}, and opting out of the program. This assumption makes the direct private effect of Medicare and Medicaid utility-neutral cohort by cohort and isolates the general-equilibrium effects of public old-age health spending operating through demand.[29] Finally, we set the public health R&D subsidy in 1940 to $s_{h,1940} = 0.2$ to match the size of the health-related R&D program (CMR) relative to private biomedical research spending prior to the program in 1940.[30]

Initial Conditions. We initialize the model in 1780 such that 1820 is the first period we solve for with initial state variables $(q_{c0}, q_{h0}, T_1) = (q_{c1780}, q_{h1780}, T_{1820})$. We keep track of previous sectoral qualities to compute current sectoral qualities according to the law of motion (21), and transfers from the deceased are required to compute current CAH of the young. We initialize the consumption sector quality, q_{c0}, such that initial income makes households indifferent to surviving into the second period (no-suicide condition) pinned down by $0 = \beta\psi_0 u(Rs_1)$. We initialize the health sector quality, q_{h0}, to match the modern health output share in 1940, conditional on matching the take-off timing in 1940.[31] Given the survival rate in phase 1, ψ_0, transfers in period 1 are determined by initial savings, $T_1 = Rs_0(1-\psi_0)$. We initialize savings such that the small open economy has a zero net foreign asset position in period 1 so that $k_1 = s_0$.

[28] Medicare is from the CBO's Historical Budget Data workbook, sheet "5a. Mandatory Outlays (GDP)". The Medicaid nursing-home component is from CMS Historical National Health Expenditure Accounts, table "National Health Expenditures by type of service and source of funds, CY 1960–2024," category "Nursing Care Facilities and Continuing Care Retirement Communities."

[29] Formally, b_t^{MC} solves the indifference condition $U_t(x_t^{MC}, p_t;\; b + b_t^{MC}) = U_t(x_t^{MC} + \tau_t^{MC}, p_t;\; b)$ for each generation t. Given $\sigma > 1$, b_t^{MC} then converges to zero along the asymptotic BGP as the consumption term $c^{1-\sigma}/(1-\sigma)$ vanishes. Accordingly, the long-run characterization in Section 4.3 and Section B.3 is unchanged and is written with the constant value b for convenience.

[30] The CMR budget was about $25 million between 1941 and 1945, or $5 million per year on average, see Gross and Sampat (2025) and Ladimer (1954). Industry biomedical research spending was around $25 million in 1940 according to Bloom and Randolph (1990).

[31] As described in section 4.1, we target the kickoff timing of modern health by initializing the health quality sufficiently low to ensure that any price consistent with labor market clearing (43) is above households' willingness to pay (40) prior to 1940 but sufficiently high to ensure an equilibrium in 1940. Within that range, we calibrate q_{h0} to match the 1940 modern health output share (see section 2).

Table 3: Internal Calibration

	Parameter		Moment	Data	Model
Households					
b	Base value of life	2.66	VSL in 2000 (millions of 2000 dollars)	3	3.10
$1/\sigma$	IES	0.78	BGP condition: $\sigma = \xi + 1$		
Health investment					
ν	Non-homotheticity	0.04	Remaining LE in 1820	40.44	40.44
χ	Modern health weight	0.90	Remaining LE in 1860	41.57	41.56
ρ	CES curvature	0.52	Increase in health output share 1940-1980	6.72pp	6.87pp
ξ	Survival function curvature	0.28	Mortality elasticity from Hall and Jones (2007)	0.16	0.16
Production and R&D					
λ	Innovation step size	10.54	Annual GDPpc growth from Maddison since 1820	1.53%	1.46%
$q_{c,0}$	Initial quality cons	1050.29	Initial no-suicide condition		
$q_{h,0}$	Initial quality health	0.38	Health output share in 1940	2.14%	2.14%

6 Results

6.1 Model Validation: How (Well) Does the Model Make Sense of the Facts?

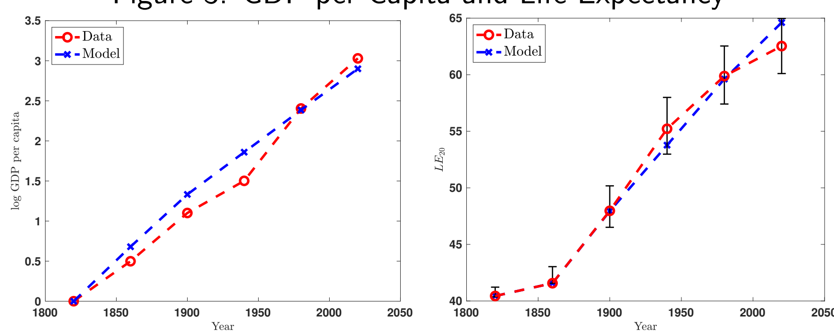
Panel (a) of Figure 3 displays the time series of real GDP per capita in the model and in the data, both plotted in natural log-scale. Per capita income growth in the model is endogenous and reflects innovation-driven growth in sectoral qualities (q_{ct}, q_{ht}), initially only in the final goods sector while the health sector is inoperative, and after 1940 also driven by innovation and thus quality growth in the modern health sector. We observe that the model matches the roughly linear path of log income per capita in the data well, driven by an aggregate postwar R&D intensity of about 4.2% of GDP which is broadly in line with U.S. data.[32] We view this as a validation of the innovation process as R&D intensity is untargeted.

Panel (b) of Figure 3 displays our estimated time series of U.S. cohort life expectancy at age 20 from the data and from the model. The 1820 and 1860 observations are directly targeted in the calibration, while the subsequent path is untargeted. Note that the estimated cohort-life-expectancy series (Data) for year 1980 is to a large extent and for year 2020 almost exclusively based on predicted mortality rates. We observe that

[32] Measured U.S. domestic R&D was 3.4% of GDP in 2020 (National Center for Science and Engineering Statistics, 2022) while the broader BEA measure of intellectual property products investment (consisting of R&D; entertainment, literary, and artistic originals; and software) was 5.8% of GDP in 2020 (Fixler and de Francisco 2022).

the model, on account of endogenous income growth and unbalanced endogenous growth between the final goods sector and the emergence of the modern health sector in the 1940s, implies that life expectancy grows continually throughout the last 2 centuries and matches the data on cohort life expectancy well. In particular, adult life expectancy first starts to rise during the nineteenth century and increases before the modern health sector emerges, driven by basic health investments. The continued growth after the health sector emerges then reflects both modern and basic health investment.

Figure 3: GDP per Capita and Life Expectancy



(a) GDP per Capita Index (Log Scale) (b) Remaining Life Expectancy

Notes: Natural logarithm of real GDP per capita index and remaining cohort life expectancy in model and data. *Source:* Data sources as in Figure 1, own calculations.

The output-, labor- and R&D shares underlying the expansion of the modern health sector since its emergence in 1940 are shown in Table 4. The 1940 output share and its increase between 1940 and 1980 are targeted, while the employment and R&D shares are untargeted. The model reproduces the increasing share of economic activity accruing to the modern health sector.[33] It overpredicts the health employment share which the parsimonious two-sector structure cannot separately target, but which we do not view as a key object of the paper. The fit is better for spending and R&D, which determine the relative quality and thus price of health over time.

We next ask how well the model fits the data on the relative price of modern health goods, which is untargeted in the calibration, and how the cost of modern health treatments has evolved over time relative to consumption goods through the lens of the model. The relative health price provides validation of the model's account of the health expansion as the health sector is disciplined by nominal health spending, not real spending

[33] As in many macroeconomic models of health spending, e.g., Hall and Jones (2007), Jones (2016) and Babalievsky (2026), our calibrated model features income elasticities of health spending above one along the transition (converging to one in the BGP).

Table 4: Modern Health Shares: Output, Employment, R&D

	Output			Employment			R&D Spending		
	1940	1980	2020	1940	1980	2020	1940	1980	2020
Data	0.021	0.089	0.176	0.014	0.056	0.104	-	0.13	0.173
Model	0.021	0.090	0.157	0.025	0.103	0.179	0.036	0.123	0.206

Sources: Data sources as in Figure 2, own calculations, R&D spending refers to the industry plus government measure.

and prices. Thus, matching the untargeted path of the relative price indicates whether the model is decomposing the observed rise in health spending into price versus quality movements in a plausible way. Moreover, aggregate quality-adjusted health price series are difficult to construct empirically, and the model provides a disciplined lens on the relative quality improvements required to reconcile spending and observed medical prices. The BEA health price index (as shown in Section 2) has no explicit adjustment for quality improvements in the medical services and goods underlying it and we interpret it as capturing non-outcome-adjusted market prices for medical services, rather than a quality-adjusted price index for health improvements, in line with the literature.[34] To the extent that the quality of modern health goods has improved, it is therefore too high relative to the model price p_t, which is the quality-adjusted relative price of health. How do we measure the model analogue price that accounts for the lack of quality adjustment and can be compared to the BEA price index? To see this, consider the production of X-rays that consist of a certain number of pixels such that the real health units are the number of pixels. Our model price p_t is the price per pixel, that is, per quality-adjusted unit. Assume next, that one machine and one worker are required to produce one X-ray. Then, in any given period, by our aggregate sector specific production function (10b), the X-ray consists of $q_{ht}^{1-\alpha_h}$ pixels and thus the quality of an X-ray increases with $q_{ht}^{1-\alpha_h}$. The BEA health price index, which measures the price of an X-ray at any point in time regardless of its pixel content, is therefore best compared to $p_t q_{ht}^{1-\alpha_h}$, which we refer to as the non-quality-adjusted price. In Table 5 we compare the BEA health price index with the quality-adjusted price p_t and the non-quality-adjusted price, $p_t q_{ht}^{1-\alpha_h}$ from the model. The

[34] See, e.g., Cutler et al. 2022 and Dunn et al. 2022. The BEA price index is built from BLS medical CPI/PPI series. For the CPI medical components, the BLS documentation lists the quality-adjustment method as "none", and the BEA applies no further quality adjustment. For the BLS medical PPIs, pricing is based on a fixed-definition service (for example, a given type of hospital stay or physician visit), which contains some cost-based adjustments when specifications change, but likewise no explicit outcome-based quality adjustment for improvements in treatment effectiveness or patient health.

Table 5: Health Price Index

	1940	**1980**	**2020**
Data	1.0	1.66	3.18
Model: Non-Quality Adjusted Price $p_t q_{ht}^{1-\alpha_h}$	1.0	1.66	2.71
Model p_t	1.0	0.33	0.13

Sources: Data sources as in Figure 2, own calculations.

non-quality-adjusted price is increasing, closely tracking the observed price in the data. Thus, despite only targeting nominal spending on modern health goods in the calibration, the model does well in separately matching the evolution of the price and quantity of health goods over time. Focusing on the quality-adjusted health price p_t, which is not directly observable in the data, our results show the price of one quality-adjusted unit of health goods is falling over time, implying that improving longevity by means of the modern medical sector has become cheaper over the last 80 years relative to the price of consumption. Through the lens of the model, relative quality improvements in the health sector reconcile the observed large increase in relative medical prices as measured by the BEA index with a substantial decline in the quality-adjusted price of modern health goods, in line with Pretnar and Feldman (2025) and Babalievsky (2026).

Is the Magnitude of the Decline in the Quality-Adjusted Price Plausible? We can compare the model-implied decline in the quality-adjusted relative health price p_t to existing outcome-based estimates of quality-adjusted medical prices. Table 6 reports the annualized growth rates of the relative health price in the model, showing a slowing decline from 2.76% per year between 1940 and 1980 to 1.67% per year over the coming 40 years. Most comparable to our aggregate price index are Cutler et al. (2022), who estimate a

Table 6: Change in the model-implied quality-adjusted relative price of health, p_t.

	Annualized growth rate (%)		
Period	1940–1980	1980–2020	2020–2060
Value	-2.76	-2.25	-1.67

2.4% annual decline in a quality-adjusted medical price index for the U.S. elderly over 1999–2012, as well as Dunn et al. (2022), who construct an aggregate medical cost-of-living index from more than 8,000 cost-effectiveness studies and document that it fell by about 1.3% per year relative to economy-wide inflation over 2000–2017. Both estimates are quantitatively consistent with the decline implied by our model.

Is the Magnitude of the Core Mechanism Empirically Plausible? The key mechanism through which our model explains the rise in life expectancy is household income growth that leads to a rise in health investments, in the presence of a decline in the (quality-adjusted) price of modern health goods. We calibrated the model in such a way that it is consistent with the rise in health spending and the phases of the health transition observed in the time series. As a further check that our model gets the income-to-life expectancy effect right quantitatively, in Appendix C.6 we compute the model-implied income-LE gradient in the cross-section and contrast it here with the empirical estimates from Chetty et al. (2016). These authors report that moving an individual up 5 percentiles in the cross-sectional income distribution (from the 45th percentile to the median, say) leads to an increase in life expectancy by 0.7-0.9 years. They make clear that these estimates are correlations rather than causal effects since unobserved confounding factors could drive both income and life expectancy.

In our model, income growth, in partial equilibrium, is the only driver of the rise in life expectancy. To determine the cross-sectional income-LE gradient, we create an artificial cross-section of households with different incomes in the model, corresponding to the mean incomes in quintiles Q2, Q3, and Q4 of the empirical income distribution, and compute their life expectancy at the current equilibrium modern health price.[35] In Table 7, we report for the 1980 and 2020 cohorts the resulting cross-sectional life expectancy levels and the implied average increase in life expectancy for each 5-percentile step. We

Table 7: Model-implied remaining life expectancy at age 20 across income quantiles.

Cohort	Q2	Q3	Q4	LE per 5-percentile step
1980	56.55	59.61	61.71	0.65
2020	62.08	64.61	66.55	0.55

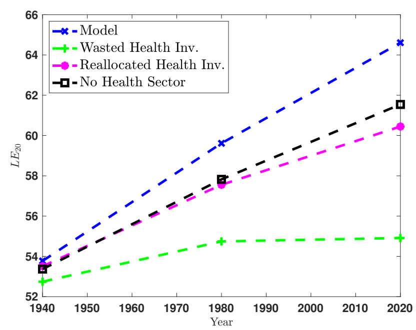
find an average increase in life expectancy of 0.65 and 0.55 years for every 5-percentile increase in the income distribution for the 1980 and 2020 cohorts, respectively, somewhat lower but in the ballpark of the gradient reported by Chetty et al. (2016). Note, however, that in our model there are no confounding cross-sectional factors (and in this sense the model cross-sectional income-LE gradient has a causal interpretation), and therefore we would expect our gradient to be lower.

[35] Details of this thought experiment are contained in Appendix C.6

6.2 Decomposing Life Expectancy

We now use our model to quantify the importance of the emergence of the modern health sector for the improvement in life expectancy in the last 80 years. In Figure 4 we plot remaining life expectancy at age 20, for the benchmark model, and in a sequence of counterfactual thought experiments to quantify the importance of the modern health sector for the increase in life expectancy from 1940 onward. The upshot of the figure is that modern health goods have been a quantitatively important driver of the 10.8 year rise in life expectancy between 1940 and 2020, but that income growth and the associated increase in basic health spending (better, more balanced diet and hygiene) would have driven up life expectancy significantly even in the absence of a modern health sector.

Figure 4: Decomposing Remaining Cohort Life Expectancy 1940-2020



Notes: The figure shows remaining life expectancy in the benchmark model (blue); in a no-reoptimization counterfactual that renders modern health spending ineffective (green); in a counterfactual that reallocates modern health spending $p_h i_h$ to basic health spending i_c while keeping total health expenditure fixed (pink); and in a one-sector counterfactual in which the modern health sector never emerges and households reoptimize having access only to basic health goods (black).

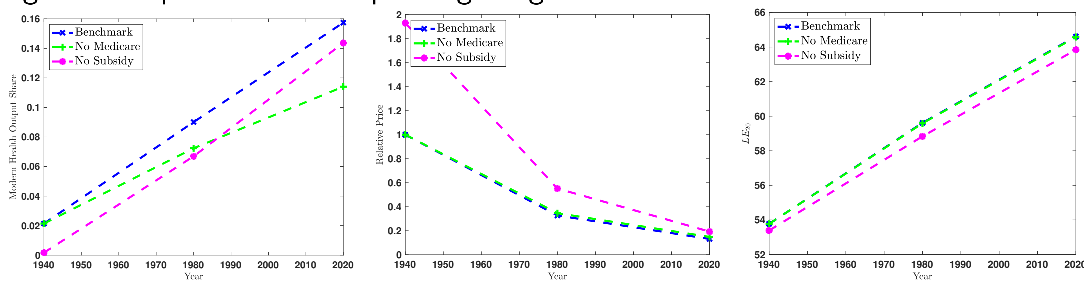
To make this point, the most extreme version of abstracting from the importance of the modern health sector is the green line "Wasted Health Investment". It renders modern health expenditures completely ineffective without allowing households to re-optimize, leaving the income-growth induced expansion of basic health spending as the only source of the mortality reduction. The impact of the basic health expansion alone is very modest, resulting in an improvement in residual life expectancy of only 2.2 years between 1940 and 2020, compared with 10.8 years in the benchmark, suggesting that the modern health sector accounts for approximately 80% of postwar life expectancy gains. Assuming that all expenditures in the modern health sector are wasted to assess the importance of this sector might be too extreme. The figure presents two alternatives, by assuming either that all modern health spending in a given period, $p_t i_{ht}$, is reallocated to basic health spending, i_{ct} ("Reallocated Health Investment"), and by considering and resolving a counterfactual one-sector economy without modern health thereby permitting households to reoptimize the level of health expenditures over time ("No Health Sector"). When resolving the one-sector economy, we retain the benchmark calibration of the model.

Reallocating expenditures expresses, in units of life expectancy, how substitutable the two types of health goods are. Compared to the case of wasted health expenditures, more than half of the loss in life expectancy by 2020 is offset when expenditures are reallocated to basic health expenditures. Yet, a gap of about 4.2 years remains, reflecting the distinct contribution of modern health technologies to mortality improvements that cannot be replicated by spending on basic health alone at equal cost. When households are able to reoptimize their basic health spending in a one-sector economy, they spend a higher share of their income on basic health goods and enjoy higher income overall because no resources are allocated to the less productive health sector. Remaining life expectancy is 3.1 years lower by 2020 and postwar life expectancy gains are almost 25% lower relative to the benchmark economy. We view this as our most plausible estimate of the contribution of the modern health sector to postwar life expectancy gains.

6.3 The Importance of Public Spending for Modern Health

How important has public spending been for the emergence and growth of the modern health sector, and life expectancy since 1940? We study this question by shutting down the two public spending programs one by one and resolving the resulting counterfactual economies, keeping the benchmark calibration fixed. In panel (a) of figure 5 we plot the health output share for all three economies. Through the lens of the model, government spending on research and development in medical and health-related fields during World War II has been crucial to the emergence of the modern health sector in the 1940s and, because it operates through innovation, also for its expansion thereafter through its persistent effects on productivity. Without public R&D, the modern health sector would have barely gotten off the ground, with a health spending share of only 0.2% in 1940 and about 1.4 percentage points lower in 2020. Panel (b) of figure 5 also makes this point in terms of the health price and panel (c) in terms of remaining life expectancy. It reveals that R&D subsidies during World War II, by raising R&D and therefore productivity in the health sector, q_{ht}, drove down health prices. Without the R&D program, health goods would have been almost twice as expensive in 1940 and 50% more expensive today. Households respond to higher prices by substituting away from modern health goods

Figure 5: Impact of Public Spending Programs on the Health Transition in the U.S.



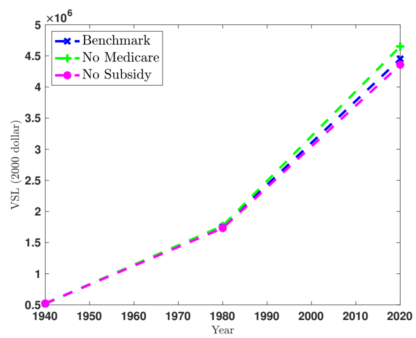
(a) Health Output Share (b) Health Price p_t (c) Life expectancy

towards basic health goods and consumption. As a result, R&D subsidies during WWII account for 0.8 years of remaining life expectancy by 2020 through the lens of our model.

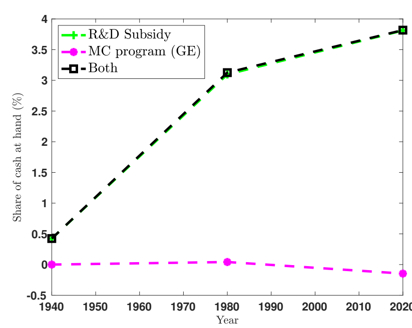
Public spending on health of the old through Medicare and nursing home expenditures through Medicaid, starting in 1980 in the model, has been an important growth driver of the modern health sector by raising demand for health goods. We find that without public old-age health spending through the two programs only about 11% of total spending would go to modern health goods in 2020, about two thirds of the current modern health spending share, and medical prices would be 13% higher, driven by reduced innovation incentives. Turning to life expectancy, recall that public health spending on the old does not directly affect survival in our model but only through its equilibrium effects on income and prices. Public health spending reallocates economic activity to the relatively less productive health sector which reduces overall productivity and income. Thus medical prices and income are both higher in the absence of Medicare and Medicaid, and we find that the net effect on households' budget constraint and life expectancy is negligible.

Finally, Figure 6 reports the model-implied VSL in constant 2000 dollars and welfare. Recall that the value-of-life parameter b is calibrated to match a VSL of $3 million in 2000. The VSL is rising over time as old-age consumption and thus utility increases, and the marginal utility of young consumption falls. In constant 2000 dollars, it reaches about $4.5 million for the current 2020 cohort, which translates to about $6.7 million in current dollars, broadly in line with existing empirical estimates for the value of a statistical life (see e.g. Viscusi and Aldy (2003), Cordoba and Ripoll (2017), Hall and Jones (2007)). The value of a statistical life rises by about 4.5% for the 2020 cohort when we shut down Medicare and Medicaid, driven by higher income. Thus, households privately achieve higher welfare in old age by reallocating the additional income towards health and old-age consumption than the additional value of being alive, b_t^{MC}, from public health spending. In contrast, when we shut down the 1940 health R&D subsidy, the VSL drops by about 2% for the 2020 cohort. Regarding the welfare consequences of the reforms, we compute the required compensation in terms of cash-on-hand to make a newborn household in a given year indifferent between being born into an economy without government subsidies (respectively without Medicare and Medicaid) and into the benchmark economy with the respective government programs in place. Thus, a positive number means that newborn households benefit from the program. Note, that the counterfactual economy to which we compare is one in which the respective program never existed and will never exist, thus it has a different price and different state variables in any given period. As shown in panel (b) of Figure 6 we find strong welfare benefits from the R&D subsidies for the household born in year 2020 of almost 4% of cash-on-hand. In contrast, conditional on our utility-neutral calibration of the program's individual old-age benefits, public health spending on the old slightly lowers welfare for the 2020 cohort by reallocating resources to the relatively less productive sector and thereby reducing equilibrium wages.

Figure 6: Implications for the Value of a Statistical Life and Welfare



(a) Value of Statistical Life



(b) Welfare

6.4 The Long Run

Where will the economy stand in the very long run and how far are we off the balanced growth path today? Table 8 provides the answer to this question. On the BGP, remaining cohort life expectancy at age 20 is at 80 years and it will take until 2126 to reach a remaining life expectancy of 72 years, i.e. to close the remaining gap between the 2020 and BGP value by 50% (half-time). As the relative price of modern health goods, p_t and thus of overall health, $P(p_t)$, continues to fall towards the BGP, households reallocate health spending from basic towards modern health goods since the two goods are relatively substitutable, while reallocating overall spending from health towards consumption since the two are complements. As a result, household spending on health as a share of cash

Table 8: Transition to the Balanced Growth Path

	1940	2020	BGP	2020 / BGP	Half-time (kickoff)	Half-time (2020)
LE at 20	53.8	64.6	80	0.8	1985	2126
Household health spending in cash (%)	21.5	24.6	18.0	1.37	1885	2130
Basic health	18.5	8.2	0.4	18.72	1826	2064
Modern health	3.0	16.4	17.6	0.93	1971	2030
Modern health output share (%)	2.1	15.7	19.4	0.81	1984	2049
Household	2.1	11.4	10.6	1.08	1964	2420
Public	0.0	4.3	8.8	0.49	2023	2121
Modern health employment share (%)	2.5	17.9	21.9	0.82	1983	2049
Modern health R&D share (%)	3.6	20.6	15.8	1.31	1960	2284
Relative price of modern health	1	0.131	0.009	15.38	–	2058

Notes: Long-run transition and balanced growth path statistics of the model. Half-time denotes the year in which half of the distance to the BGP value is closed, either from the baseline value prior to kickoff of the corresponding variable (e.g. 0% for the health spending measures) or its value in 2020.

at hand stabilizes at 18% in the long run, lower than it is today, and almost all of this spending is on modern health goods. Focusing on the aggregate economy, the modern health sector will account for more than 19% of total production, rising from its current value of almost 16%. Thus, today the modern health sector has reached about 80% of its long-run size, and its long-run transition is rapid with a half-time of less than 30 years.

7 Conclusion

We build a quantitative theory of income growth, the increase in life expectancy in the last two centuries, and the emergence and expansion of a modern health sector in the 20th century. Our two-sector OLG model with endogenous directed technical change endogenously determines income growth, life expectancy, and technological progress in the health sector and the final goods sector, as well as the size of the health sector and the quality and price of the goods in equilibrium. We show that it can generate an economic path in which households are initially poor and the quality-adjusted price of health goods is prohibitively high so that demand for health goods is zero, life is short and life expectancy is stagnant. As income grows, fueled by technological progress, households start consuming basic health goods, life expectancy starts to rise, and directed technological progress eventually, with a delay of ca. 100 years, leads to the emergence and expansion of a modern health sector. Through the lens of the model, the quality-adjusted relative price of health goods declined by about 2.5% per year between 1940 and 2020, driven by faster productivity growth in the modern health sector than in the broader economy. We employ the model to quantify the role of the modern health sector for the evolution of life expectancy. In our preferred counterfactual in which the modern health sector never emerges, modern health accounts for 25% of postwar life expectancy gains. Finally, we investigate the role of government spending for the expansion of modern health. We find sizable positive welfare effects of R&D subsidies during World War II, without which the modern medical sector would have barely gotten off the ground in 1940 and remaining life expectancy in 2020 would be about 0.8 years lower. In contrast, public health spending on the old through Medicare and Medicaid, while expanding demand for modern health goods, reduced income per capita by reallocating activity toward the relatively less productive health sector and thereby slightly reduced welfare.

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Supplementary Online Appendix

A Data Appendix

A.1 Life Expectancy

Our data to construct cohort life expectancy comes from three different sources. First, we use gender specific historical mortality rates for the US that were originally collected and imputed by Haines (1994), which were updated by Hacker (2010). This data covers the time period 1790 to 1899 and comes at a decennial frequency for the age groups $\{0, 1-4, 5-9, \ldots, 80\}$, where the authors report mortality rates of 1 for age 80 onward. We carry out two key transformations to this data, whereby we suppress the gender index for ease of presentation[36]: (i) we hold mortality rates constant within age group and compute age specific mortality rates in each age group i, m_j^i such that m^i is the respective geometric average within this age group, i.e., $m_j^i = 1 - (1 - m^i)^{1/n_i}$ where $n_i \in \{4, 5\}$ denotes the width of age group i; (ii) to obtain estimates of mortality rates above (and including) age 80, we estimate per period a Gompertz–Makeham mortality model on the (constructed) age specific mortality rates and set the mortality rates for all ages 80 and older to the predicted values. Second, for the years 1900 to 1932 we use data from the Human Life-Table Database, which have age specific mortality data for ages $0, 1, \ldots, 105$. We append those to the historical mortality rates and smooth the resulting mortality rates over years 1790 to 1932 and ages 5 and older with a kernel density smoother and a bandwidth parameter of 5. Third, for the years 1933 to 2021 we use data from the Human Mortality Database with age and time specific mortality rates up to age 110.

To predict future mortality rates—required for the computation of cohort life expectancy—we estimate future trends in mortality by a Lee-Carter (Lee and Carter 1992) method decomposing the age and time specific weighted average (across genders) mortality rates into the age-specific components a_j, b_j and a single time index k_t as

$$\ln(m_{jt}) = a_j + b_j k_t.$$

[36] To compute the average (across genders) mortality rates and their prediction, which we use in the paper, in addition to female and male mortality rates, all of the following data operations are performed on the population weighted average mortality rates.

We assume a stochastic trend of the single index,

$$k_t = \alpha + \rho k_{t-1} + \epsilon_t.$$

and estimate the parameters a_j, b_j, k_t in a first stage and α, ρ in a second stage using data for the period 1933 to 2021.[37]

The baseline path of predicted mortality rates is computed using the estimated values $\hat{a}_j, \hat{b}_j, \hat{\alpha}, \hat{\rho}$ and setting to 0 the innovations ϵ_t for all years $t \geq 2021$ until our forecast horizon 2110.[38] The bootstrapped confidence intervals are computed by bootstrapping along the time dimension of the cross-sectional mortality rates for the historical data before 1933.[39] For the period 1933 to 2021, as well as for the prediction until 2110, we bootstrap by drawing from the residuals ϵ_t.[40]

[37]To smooth out the Corona mortality crises induced reduction of life expectancy, we set the mortality rates in 2020 and 2021 to their year 2019 values. Our estimation procedure decomposes a_j, b_j and k_t by a standard Lee-Carter method. We additionally scale the determined k_t-sequence to exactly match cross-sectional life expectancy.

[38]For calibration purposes, we need a time sequence of remaining cohort life expectancy of 20 year old persons up to year 2020 (i.e. for a person of age 20 in year 2020). Human Mortality Database data range to age 110, a person of age 20 in year 2020 lives at most until year 2110.

[39]We implement the bootstrap procedure as a block bootstrap and set the width of the blocks according to the standard rule of thumb $bw = T^{1/3}$. Since the historical data span $T = 143$ years, this gives $bw = T^{1/3} = 5.23$, of which we take the ceil.

[40]These residuals are not autocorrelated, thus there is no need to apply a block bootstrap approach.

B Model Appendix

B.1 Analysis of Firm Problems and Aggregation: Proof of Proposition 1

From equation (7) in the main text and the production function for intermediates (which is linear in capital) we immediately obtain that

$$\frac{y_{jit}}{q_{jit}} = \frac{k_{jit}}{q_{jit}} = \left(\frac{p_{jt}\alpha_j^2}{R}\right)^{\frac{1}{1-\alpha_j}} l_{jt} \tag{B.1}$$

and, thus, the choice of each intermediate good is proportional to its corresponding quality q_{jit}. Aggregating across intermediates i immediately delivers

$$k_{jt} = \int k_{jit} di = \left(\frac{p_{jt}\alpha_j^2}{R}\right)^{\frac{1}{1-\alpha_j}} l_{jt} \int q_{jit} di = \left(\frac{p_{jt}\alpha_j^2}{R}\right)^{\frac{1}{1-\alpha_j}} l_{jt} q_{jt}$$

which is equation (10a) in the main text. Equation (10b) follows from the aggregate production function (1) and the fact that

$$y_{jt} = \left(\int_0^1 q_{jit}^{1-\alpha_j} y_{jit}^{\alpha_j} di\right) l_{jt}^{1-\alpha_j} = \left(\int_0^1 q_{jit} \left(\frac{k_{jit}}{q_{jit}}\right)^{\alpha_j} di\right) l_{jt}^{1-\alpha_j} = (k_{jt})^{\alpha_j} (q_{jt} l_{jt})^{1-\alpha_j}$$

Equation (11a) follows immediately from the first order condition for labor l_{jt}, by multiplying and dividing the left-hand side of equation (3) by l_{jt}. To derive equation (11b), replace y_{jit} by k_{jit} in (7) (since intermediate goods production is linear in capital input), and then aggregate:

$$k_{jt} = \int k_{jit} di = \left(\frac{p_{jt}\alpha_j^2}{R}\right)^{\frac{1}{1-\alpha_j}} l_{jt} \int q_{jit} di = \left(\frac{p_{jt}\alpha_j^2}{R}\right)^{\frac{1}{1-\alpha_j}} l_{jt} q_{jt}$$

Exponentiating both sides by $(1-\alpha_j)$ and solving for R delivers

$$R = p_{jt}\alpha_j^2 \frac{(l_{jt}q_{jt})^{1-\alpha_j} k_{jt}^{\alpha}}{k_{jt}} = \alpha_j^2 \frac{p_{jt} y_{jt}}{k_{jt}}$$

as stated in equation (11b). Equation (10c) then follows from aggregating equation (4), using (8):

$$\pi_{jt} = \int \pi_{jit} di = \frac{1-\alpha_j}{\alpha_j} R \int y_{jit} di = \frac{1-\alpha_j}{\alpha_j} R k_{jt} = (1-\alpha_j)\alpha_j p_{jt} y_{jt}.$$

Finally, equation (12) follows directly from equations (10c)-(11b).

B.2 Analytical Solution to the Household Problem

B.2.1 Division of Health Investment

In this subsection we analyze the division of health expenditures between modern health goods and basic health goods, and provide a proof of Proposition 3.

Part (1) of the proposition for $e_t = 0$ is trivial. Now assume $e_t > 0$. The basic strategy to prove the remainder of Proposition 3 is to first characterize an interior solution for (i_{ct}, i_{ht}), ignoring the non-negativity constraints (and thus deriving the expressions in item (3) of the proposition), and then obtain conditions under which these constraints are binding for either basic or modern health good spending.

For a given amount of health expenditures $e_t > 0$ and relative price p_t households solve

$$i_t(p_t, e_t) = \max_{i_{ct}, i_{ht} \geq 0} \left\{ \left[\chi \left(i_{ht} + \nu\right)^{\rho} + (1-\chi)\left(i_{ct} + \nu\right)^{\rho}\right]^{\frac{1}{\rho}} \right\} \quad \text{(B.2a)}$$

$$s.t. \quad p_t i_{ht} + i_{ct} = e_t \quad \text{(B.2b)}$$

Interior Solution. In an interior solution households spend on both types of health goods and the first-order conditions hold with equality. It is helpful to transform the non-homothetic CES specification into a homothetic one by letting households choose "effective health investments", $\tilde{i}_{ct} = i_{ct} + \nu$ and $\tilde{i}_{ht} = i_{ht} + \nu$, and verifying the resulting optimal choices (i_{ct}, i_{ht}) are in fact interior. For a given level of "effective health spending", $\tilde{e}_t = e_t + (1 + p_t)\nu$, and relative price p_t of modern health goods, households then solve

$$i_t(p_t, \tilde{e}_t) = \max_{\tilde{i}_{ct}, \tilde{i}_{ht} \geq \nu} \left\{ \left[\chi \tilde{i}_{ht}^{\rho} + (1-\chi)\tilde{i}_{ct}^{\rho}\right]^{\frac{1}{\rho}} \right\} \quad \text{(B.3a)}$$

$$s.t. \quad p_t \tilde{i}_{ht} + \tilde{i}_{ct} = \tilde{e}_t \quad \text{(B.3b)}$$

The Lagrangian for the problem is given by

$$\mathcal{L} = \left[\chi \tilde{i}_{ht}^{\rho} + (1-\chi)\tilde{i}_{ct}^{\rho}\right]^{\frac{1}{\rho}} - \lambda\left(p_t \tilde{i}_{ht} + \tilde{i}_{ct} - \tilde{e}_t\right), \tag{B.4}$$

for which the two first-order conditions are

$$0 = \left[\chi \tilde{i}_{ht}^{\rho} + (1-\chi)\tilde{i}_{ct}^{\rho}\right]^{\frac{1}{\rho}-1} \chi \tilde{i}_{ht}^{\rho-1} - \lambda p_t,$$
$$0 = \left[\chi \tilde{i}_{ht}^{\rho} + (1-\chi)\tilde{i}_{ct}^{\rho}\right]^{\frac{1}{\rho}-1} (1-\chi) \tilde{i}_{ct}^{\rho-1} - \lambda.$$

Combining the two conditions yields the following relationship between effective basic and modern health spending

$$\tilde{i}_{ht} = \tilde{i}_{ct}\left[\frac{\chi}{(1-\chi)p_t}\right]^{\frac{1}{1-\rho}}.$$

Plugging this relationship into the budget constraint, effective basic health spending is given by

$$\tilde{i}_{ct} = \tilde{\vartheta}(p_t)\tilde{e}_t,$$

where

$$\tilde{\vartheta}(p_t) \equiv \left[1 + \left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}} \left(\frac{1}{p_t}\right)^{\frac{\rho}{1-\rho}}\right]^{-1} \tag{B.5}$$

is the effective health expenditure share spent on basic goods.[41] The level of basic health spending is then given by, recalling that $\tilde{e}_t = e_t + (1+p_t)\nu$,

$$\begin{aligned} i_{ct}(e_t, p_t) &= \tilde{i}_{ct} - \nu \\ &= \tilde{\vartheta}(p_t)\tilde{e}_t - \nu \\ &= \tilde{\vartheta}(p_t)e_t + \tilde{\vartheta}(p_t)\nu(1+p_t) - \nu \\ &= \tilde{\vartheta}(p_t)e_t + \nu\left[\tilde{\vartheta}(p_t)(1+p_t) - 1\right], \end{aligned} \tag{B.6}$$

[41] Note that the effective share spent on basic goods $\tilde{\vartheta}(p_t)$ is strictly increasing in p_t if $\rho \in (0,1)$ and the two investments are relatively substitutable, is independent of p_t if $\rho = 0$ (the Cobb-Douglas case) and is strictly decreasing in p_t if $\rho < 0$.

and the level of modern health spending is given by

$$\begin{aligned}
p_t i_{ht}(e_t, p_t) &= p_t \tilde{i}_{ht} - p_t \nu \\
&= \Big(1 - \tilde{\vartheta}(p_t)\Big)\, \tilde{e}_t - p_t \nu \\
&= \Big(1 - \tilde{\vartheta}(p_t)\Big)\, e_t + \Big(1 - \tilde{\vartheta}(p_t)\Big)\, \nu (1 + p_t) - p_t \nu \\
&= \Big(1 - \tilde{\vartheta}(p_t)\Big)\, e_t + \nu \left[\Big(1 - \tilde{\vartheta}(p_t)\Big)(1 + p_t) - p_t\right]. \qquad \text{(B.7)}
\end{aligned}$$

This establishes part (3) of the proposition.

Corner Solution for modern health goods: A corner solution for modern health spending, $i_{ht} = 0$, is the optimal choice of the household if the solution above yields $\tilde{i}_{ht} < \nu$ (that is, $i_{ht} < 0$) which, using equation (B.7) is the case if overall health expenditures e_t are sufficiently low relative to the price of modern health goods, that is, if

$$\begin{aligned}
&\Big(1 - \tilde{\vartheta}(p_t)\Big)\, e_t < \nu \left[p_t - \Big(1 - \tilde{\vartheta}(p_t)\Big)(1 + p_t)\right] \\
\Leftrightarrow \qquad & e_t < \nu \left[\frac{p_t}{1 - \tilde{\vartheta}(p_t)} - (1 + p_t)\right] = \nu \left[p_t \frac{1}{\frac{1}{\tilde{\vartheta}(p_t)} - 1} - 1\right] \\
&= \nu \left[p_t \frac{1}{\left[1 + \left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}} \left(\frac{1}{p_t}\right)^{\frac{\rho}{1-\rho}}\right] - 1} - 1\right] \\
&= \nu \left[\left(\frac{1-\chi}{\chi}\right)^{\frac{1}{1-\rho}} p_t^{\frac{1}{1-\rho}} - 1\right] = \underline{\mathsf{e}}^h(p_t). \qquad \text{(B.8)}
\end{aligned}$$

where $\underline{\mathsf{e}}^h(p_t)$ is defined as in the proposition, and is the health expenditure threshold below which the modern health sector remains inactive. This expenditure threshold $\underline{\mathsf{e}}^h(p_t)$ depends positively on the relative price of modern health goods, p_t, that is, optimal health expenditures have to be high enough relative to the price of modern health goods for this phase to commence. As stated in the proposition, $\underline{\mathsf{e}}^h(p_t) > 0$ if and only if $p_t > \underline{(}p) = \frac{\chi}{1-\chi}$. This establishes part (2) of the proposition.

Corner Solution for basic health goods: Symmetrically, the household does not spend on basic health goods if the optimal choice yields $\tilde{i}_{ct} < \nu$ (that is, $i_{ct} < 0$). This is the case if

$$\begin{aligned}
\tilde{\vartheta}(p_t)e_t + \nu\left[\tilde{\vartheta}(p_t)(1+p_t) - 1\right] < 0 \\
\Leftrightarrow \qquad e_t &< \nu\left[\frac{1}{\tilde{\vartheta}(p_t)} - (1+p_t)\right] \\
&= \nu\left[\left[1 + \left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}}\left(\frac{1}{p_t}\right)^{\frac{\rho}{1-\rho}}\right] - (1+p_t)\right] \\
&= \nu\left[\left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}}\left(\frac{1}{p_t}\right)^{\frac{\rho}{1-\rho}} - p_t\right] \qquad \text{(B.9)} \\
&= p_t\nu\left[\left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}}\left(\frac{1}{p_t}\right)^{\frac{1}{1-\rho}} - 1\right] = \underline{\mathsf{e}}^c(p_t). \qquad \text{(B.10)}
\end{aligned}$$

where $\underline{\mathsf{e}}^c(p_t)$ is defined as in the proposition, and is the health expenditure threshold below which spending in basic health is suboptimal. The dependence of this threshold on the relative health price is more complex and depends on the elasticity of substitution. We can also express the threshold in units of modern health goods as follows:

$$\frac{e_t}{p_t} < \nu\left[\left(\frac{\chi}{1-\chi}\right)^{\frac{1}{1-\rho}}\left(\frac{1}{p_t}\right)^{\frac{1}{1-\rho}} - 1\right]. \qquad \text{(B.11)}$$

In particular, this expression captures the health expenditure threshold in units of modern health purchasing power which has to be sufficiently low for households to demand basic health goods. As stated in the proposition, $\underline{\mathsf{e}}^c(p_t) > 0$ if and only if $p_t < \underline{\mathsf{p}} = \frac{\chi}{1-\chi}$. This establishes item (2b) of the proposition and completes the proof.

B.2.2 The Level of Health Expenditures

Households derive utility from consumption in young age, c_t^y, and old age, c_{t+1}^o, they survive from the first to the second period of their life with probability ψ which depends on their investment i_t into health goods when young. Expected lifetime utility is given by

$$(1-\beta)u(c_t^y) + \beta\psi(i_t)\,u(c_{t+1}^o).$$

The maximization of the utility function is subject to the constraints:

$$c_t^y + i_{ct} + p_t i_{ht} + s_t = x_t \equiv w_{ct} n_{ct} + w_{ht} n_{ht} + T_t - \tau_t \tag{B.12a}$$

$$i_t = f(i_{ct}, i_{ht}) \tag{B.12b}$$

$$1 = n_{ct} + n_{ht} \tag{B.12c}$$

$$c_{t+1}^o = R s_t. \tag{B.12d}$$

and our functional form assumptions are spelled out in the main text.

Since optimal saving is always strictly positive, potential borrowing constraints never bind and the period budget constraints can be consolidated to the lifetime budget constraint

$$c_t^y + i_{ct} + p_t i_{ht} + \frac{c_{t+1}^o}{R} = w_{ct} n_{ct} + w_{ht} n_{ht} + T_t - \tau_t \equiv x_t. \tag{B.13}$$

Given the optimal division of health expenditures e_t and the resulting health investment function $i_t(p_t, e_t)$, we now consider the problem of allocating cash-on-hand x_t between consumption c_t^y when young, savings s_t (and thus the implied consumption when old, c_{t+1}^o) and health expenditures e_t. That is, the household now solves

$$\max_{0 \le c_t^y, e_t \le x_t} \left\{ (1-\beta) u(c_t^y) + \beta \psi\left(i_t(p_t, e_t)\right) u(R(x_t - c_t^y - e_t)) \right\}, \tag{B.14}$$

where we have used the fact that from the budget constraint $s_t = x_t - c_t^y - e_t$. Because of the assumed Inada condition, we can ignore potential corners for the choice of c_t^y; however, the optimal choice of e_t might be zero. Therefore the first order conditions with respect to (c_t^y, e_t) are

$$(1-\beta) u'(c_t^y) = \beta R \psi\left(i_t(p_t, e_t)\right) u'(R(x_t - c_t^y - e_t)) \tag{B.15a}$$

$$\psi'\left(i_t(p_t, e_t)\right) u(R(x_t - c_t^y - e_t)) \frac{\partial i_t}{\partial e_t} \le R \psi\left(i_t(p_t, e_t)\right) u'(R(x_t - c_t^y - e_t)), \tag{B.15b}$$

where the second equation holds with equality if $e_t > 0$. The first equation is the standard intertemporal Euler equation for consumption between the two periods. The second equation trades off the benefits of spending an extra unit of resources on health goods today (the left hand side) with the utility loss from one unit of foregone consumption tomorrow (the right hand side). Costs and benefits are equalized whenever health spending is strictly positive; if the costs exceed the benefits, then $e_t = 0$.

Proof of Proposition 4. Fix $p > 0$. Consider the household problem (B.14) and denote $c^o(x,e) \equiv R(x - c^y(x,e) - e)$. Evaluating the health-spending optimality condition (B.15b) at the boundary $e = 0$ yields

$$\frac{d}{de}\psi(i(e,p))\bigg|_{e=0} u(c^o(x,0)) \;\leq\; R\,\psi(i(0,p))\,u'(c^o(x,0))\,, \tag{B.16}$$

By (32) and (39), the marginal survival benefit at $e = 0$ is finite, $\frac{d}{de}\psi(i(e,p))\big|_{e=0} < \infty$. Moreover, for x sufficiently small, $0 < c^o(x,0) < Rx$, so $c^o(x,0) \downarrow 0$ as $x \downarrow 0$, and the lower Inada condition implies $u'(c^o(x,0)) \to \infty$. Hence the right-hand side diverges, while the left-hand side is bounded above for x sufficiently small. Since feasible e lie in $[0,x]$, the same comparison holds uniformly over all feasible e: $\frac{d}{de}\psi(i(e,p))$ is bounded on $[0,x]$, $u(c^o(x,e)) \leq u(Rx)$, and $R\psi(i(e,p))u'(c^o(x,e)) \geq R\psi(i(0,p))u'(Rx) \to \infty$ as $x \downarrow 0$. Hence, for x small enough, (B.15b) is strict for every feasible $e > 0$, so $e(x,p) = 0$. Thus there exists $\underline{\mathrm{x}}_1(p) > 0$ such that $e(x,p) = 0$ for all $x \leq \underline{\mathrm{x}}_1(p)$. □

Proof of Lemma 1. For the first claim, fix $p > \underline{\mathrm{p}}$. For all $e \in (0, \underline{\mathrm{e}}^h(p)]$ the optimal allocation satisfies $i_h(e,p) = 0$ and $i_c(e,p) = e$ by Proposition 3. Hence

$$i(e,p) = \left[\chi\nu^\rho + (1-\chi)(e+\nu)^\rho\right]^{1/\rho},$$

so that

$$i(0,p) = \nu \qquad \text{and} \qquad \frac{\partial i(e,p)}{\partial e}\bigg|_{e=0} = 1-\chi,$$

are independent of p. The consumption allocation is also independent of p at $e = 0$, and thus condition (38b), evaluated at the extensive margin $e = 0$, is independent of p. Therefore the threshold in Proposition 4 is common across all $p > \underline{\mathrm{p}}$.

For the second claim, first suppose $p > \underline{\mathrm{p}}$. If private demand for modern health goods is positive, Proposition 3 implies that $e(x,p) > \underline{\mathrm{e}}^h(p)$. Since feasibility implies $e(x,p) \leq x$, it follows that

$$x > \underline{\mathrm{e}}^h(p) = \nu\left[\left(\frac{p}{\underline{\mathrm{p}}}\right)^{\epsilon} - 1\right].$$

Rearranging yields

$$p < \underline{\mathrm{p}}\left(1 + \frac{x}{\nu}\right)^{1/\epsilon} \equiv \bar{p}(x).$$

If instead $p \le \underline{\mathrm{p}}$, positive private demand for modern health goods implies $x > 0$, and thus $\bar{p}(x) > \underline{\mathrm{p}} \ge p$. Then in all cases positive private demand for modern health goods implies $p < \bar{p}(x)$. □

B.3 Analysis of the Balanced Growth Path

B.3.1 Household Problem in the BGP: Proof of Proposition 5

Proof of Proposition 5. Suppose there exists an asymptotic BGP with constant health price p^* and interior health spending $e_t/x_t \in (\epsilon_e, 1-\epsilon_e)$ for some small $\epsilon_e > 0$ as $t \to \infty$. Along the BGP with $x_t \to \infty$, we have $i_t = \frac{e_t + (1+p^*)\nu}{P(p^*)}$ and thus $i_t/x_t \to e_t/x_t/P(p^*) \in (\epsilon_i, 1-\epsilon_i)$ for some small $\epsilon_i > 0$. From the Euler equation (38a), and using $\psi(i_t) \to 1$, we further have

$$\frac{c^o_{t+1}}{c^y_t} \to \left(\frac{\beta R}{1-\beta}\right)^{1/\sigma}. \tag{B.17}$$

Together with the budget constraint, this then yields interior young consumption, $c^y_t/x_t \in (\epsilon_c, 1-\epsilon_c)$ for some small $\epsilon_c > 0$, and saving $s_t/x_t \in (\epsilon_s, 1-\epsilon_s)$ for some small $\epsilon_s > 0$. Dividing the BGP health-spending optimality condition (46) by $i_t^{\xi+1}$ yields

$$b\left(\frac{c^o_{t+1}}{i_t}\right)^{\sigma} i_t^{\sigma-(\xi+1)} + \frac{1}{1-\sigma}\left(\frac{c^o_{t+1}}{i_t}\right) i_t^{-\xi} = \frac{P(p^*)R}{\xi}\frac{1+i_t}{i_t}\frac{(1+i_t)^{\xi}-1}{i_t^{\xi}}. \tag{B.18}$$

The right-hand side converges to the strictly positive finite limit $\frac{P(p^*)R}{\xi} \in (0,\infty)$. Since $c^o_{t+1}/i_t = Rs_t/i_t$ is bounded away from zero and infinity, the second term on the left converges to zero because $i_t \to \infty$ and $\xi > 0$, while the first term on the left converges to a strictly positive finite limit if and only if $\sigma - (\xi+1) = 0$, which proves necessity.

To prove the converse, assume $\xi = \sigma - 1$. Using $c^o_{t+1} = Rs_t$ and the fact that in the interior allocation of Proposition 3 we have $\partial i_t/\partial e_t = P(p^*)^{-1}$, the optimality condition

(46) yields

$$b\left(\frac{Rs_t}{i_t}\right)^{\sigma} \to \frac{P(p^*)R}{\xi} \tag{B.19}$$

Then using $i_t/e_t \to \frac{1}{P(p^*)}$ and $e_t \to \infty$ along the asymptotic BGP yields

$$\left(\frac{1-\vartheta_{e,t}}{\vartheta_{e,t}}\right)^{\sigma} \to \frac{(P(p^*)R)^{1-\sigma}}{b\xi}, \tag{B.20}$$

where $\vartheta_{e,t} \equiv \frac{e_t}{e_t+s_t}$ is the health spending in total "saving" $e_t + s_t$. Then $\vartheta_e^* \in (0,1)$ is constant and interior and solving equation (B.20) for the share ϑ_e^* directly gives (47a) in the proposition. Noting that $c_t^y = \vartheta_c^* x_t$ and $c_{t+1}^o = Rs_t = R(1-\vartheta_c^*)(1-\vartheta_e^*)x_t$, and using this in the consumption Euler equation and solving for ϑ_c^* yields (47b). Thus if $\xi = \sigma - 1$, the household allocation must converge to these constant and interior shares for any asymptotic BGP with constant health price $p^* > 0$. □

B.3.2 Labor Markets in the BGP: Proof of Proposition 6

Proof of Proposition 6. Evaluating the wage-equalization condition (42) at the BGP yields

$$1 = (p^*)^{\frac{1}{1-\alpha_h}} C(\alpha_h, \alpha_c, R) r_q^*, \tag{B.21}$$

from which equation (49) in the proposition immediately follows. Since along an interior BGP, qualities in both sectors have to grow at the same rate for their ratio to be constant, in the BGP the growth rate of qualities has to be identical across both sectors, $\mu_h^* = \mu_c^*$, which then implies, using equation (19), that the relative labor allocation satisfies equation (50) in the proposition. Lastly, the ratio of the values of outputs in both sectors (51) follows directly from equation (11a) and the equalization of wages across sectors, and $S(p^*)$ in equation (53) follows from direct calculations. □

B.3.3 Health Goods Market in the BGP

We first show that aggregate demand for health goods, as a fraction of GDP, in the BGP is given by equation (48) in the main text. Then we prove Proposition 7 in the next subsection. Dividing total demand for modern health goods by GDP y_t yields:

$$\begin{aligned} D(p^*) &\equiv \frac{p_t i_{ht} + \tau_t^{MC}}{y_t} = \frac{p^* i_{ht}}{x_t}\frac{x_t}{y_t} + \hat{\tau}^{MC} = \frac{p^* i_{ht}}{e_t}\frac{e_t}{x_t}\frac{x_t}{y_t} + \hat{\tau}^{MC} \\ &= \frac{p^* i_{ht}}{e_t}\frac{e_t}{e_t + s_t}\frac{e_t + s_t}{x_t}\frac{x_t}{y_t} + \hat{\tau}^{MC} = \frac{p^* i_{ht}}{e_t}\frac{e_t}{e_t + s_t}\frac{x_t - c_t^y}{x_t}\frac{x_t}{y_t} + \hat{\tau}^{MC} \\ &= \frac{p^* i_{ht}}{e_t}\frac{e_t}{e_t + s_t}\frac{x_t - c_t^y}{x_t}\frac{x_t}{y_t} + \hat{\tau}^{MC} \end{aligned}$$

Now we note that, by definition of

$$\begin{aligned} \frac{p^* i_{ht}}{e_t} &= \frac{e_t - i_{ct}}{e_t} = 1 - \tilde{\vartheta}(p^*) \\ \frac{e_t}{e_t + s_t} &\equiv \vartheta_e^*(p^*) \\ \frac{x_t - c_t^y}{x_t} &= 1 - \frac{c_t^y}{x_t} = 1 - \vartheta_c^*(p^*) \\ \frac{x_t}{y_t} &= \frac{w_{ct} n_{ct} + w_{ht} n_{ht} + T_t - \tau_t}{y_t} = \frac{(1-\alpha_c) y_{ct} + (1-\alpha_h) p_t y_{ht}}{y_t} - \hat{\tau}^{MC} \\ &= \frac{(1-\alpha_c)(y_{ct} + p_t y_{ht}) + (\alpha_c - \alpha_h) p_t y_{ht}}{y_{ct} + p_t y_{ht}} - \hat{\tau}^{MC} = (1-\alpha_c) + (\alpha_c - \alpha_h) S(p^*) - \hat{\tau}^{MC}, \end{aligned}$$

where we used that $T_t/y_t \to 0$ as $\psi(i_t) \to 1$. Thus

$$D(p^*) = (1 - \tilde{\vartheta}(p^*))(1 - \vartheta_c^*(p^*))\vartheta_e^*(p^*) \left[(1-\alpha_c) + (\alpha_c - \alpha_h) S(p^*) - \hat{\tau}^{MC}\right] + \hat{\tau}^{MC} \tag{B.22}$$

as stated in equation (48) of the main text.

B.3.4 Existence and Uniqueness of the BGP: Proof of Proposition 7

Existence Proof of Proposition 7. Define the excess demand function of modern health goods as $E(p) \equiv D(p) - S(p)$. Using the expressions for $S(p)$ and $D(p)$ derived in the main text, excess demand is given by

$$E(p) = \hat{\tau}^{MC} + \theta(p)\left[1 - \alpha_c - \hat{\tau}^{MC}\right] - S(p)\left[1 - \theta(p)(\alpha_c - \alpha_h)\right]. \tag{B.23}$$

$S(p)$ is continuous and, since $C(\alpha_h, \alpha_c, R) > 0$, strictly decreasing with $\lim_{p \downarrow 0} S(p) = 1$ and $\lim_{p \to \infty} S(p) = 0$. Private modern health demand on the BGP is given by $\theta(p) =$

$(1-\tilde{\vartheta}(p))(1-\vartheta_c^*(p))\,\vartheta_e^*(p)$ (see Proposition 5), where

$$\tilde{\vartheta}(p) = \frac{(1-\chi)^\varepsilon}{(1-\chi)^\varepsilon + \chi^\varepsilon p^{1-\varepsilon}} = (1-\chi)^\varepsilon P(p)^{\varepsilon-1}, \tag{B.24}$$

$$\vartheta_e^*(p) = \left(1 + \left[\frac{(P(p)R)^{1-\sigma}}{b(\sigma-1)}\right]^{\frac{1}{\sigma}}\right)^{-1} \in (0,1), \tag{B.25}$$

$$\vartheta_c^*(p) = \left(1 + \left(\frac{\beta}{1-\beta}R^{1-\sigma}\right)^{\frac{1}{\sigma}} \frac{1}{1-\vartheta_e^*(p)}\right)^{-1} \in (0,1). \tag{B.26}$$

Then $\theta(p)$ is continuous with

$$\varepsilon > 1: \qquad \lim_{p\downarrow 0}\theta(p) = 0, \qquad \lim_{p\to\infty}\theta(p) = 0.$$

$$\varepsilon < 1: \qquad \lim_{p\downarrow 0}\theta(p) = 0, \qquad \lim_{p\to\infty}\theta(p) = 1.$$

Thus $E(p)$ is well-defined and continuous in p with

$$\varepsilon > 1: \qquad \lim_{p\downarrow 0}E(p) = \hat{\tau}^{MC} - 1, \qquad \lim_{p\to\infty}E(p) = \hat{\tau}^{MC}.$$

$$\varepsilon < 1: \qquad \lim_{p\downarrow 0}E(p) = \hat{\tau}^{MC} - 1, \qquad \lim_{p\to\infty}E(p) = 1 - \alpha_c.$$

Finally, for the Cobb-Douglas case $\varepsilon = 1$, we have that $1 - \tilde{\vartheta}(p) = \chi$ is constant and $P(p) = p^\chi/[\chi^\chi(1-\chi)^{1-\chi}]$. Hence $\vartheta_e^*(p) \to 0$ as $p \downarrow 0$ and $\vartheta_e^*(p) \to 1$, $\vartheta_c^*(p) \to 0$ as $p \to \infty$, so that $\theta(p)$ is continuous with $\lim_{p\downarrow 0}\theta(p) = 0$ and $\lim_{p\to\infty}\theta(p) = \chi$. Thus $\lim_{p\downarrow 0}E(p) = \hat{\tau}_{MC} - 1 < 0$ and $\lim_{p\to\infty}E(p) = \chi(1-\alpha_c) + (1-\chi)\hat{\tau}_{MC} > 0$.

Given $\alpha_c \in (0,1)$ and interior public health spending $\hat{\tau}^{MC} \in (0,1)$, we then have $\lim_{p\downarrow 0}E(p) < 0$ and $\lim_{p\to\infty}E(p) > 0$ and, by the intermediate value theorem, there exists $p^* > 0$ with $E(p^*) = 0$, i.e. $S(p^*) = D(p^*)$. □

Uniqueness Proof of Proposition 7. Differentiating E with respect to p yields

$$E'(p) = \theta'(p)\left[(1-\alpha_c-\hat{\tau}^{MC}) + (\alpha_c-\alpha_h)S(p)\right] + S'(p)\left[\theta(p)(\alpha_c-\alpha_h) - 1\right] \tag{B.27}$$

Then $S'(p)\left[\theta(p)(\alpha_c-\alpha_h)-1\right] > 0 \;\forall\; p \in (0,\infty)$ since $S'(p) < 0 \;\forall\; p \in (0,\infty)$ and $[\theta(p)(\alpha_c-\alpha_h)-1] < 0 \;\forall\; p \in (0,\infty)$. Next assume $\hat{\tau}^{MC} < 1 - \max\{\alpha_c, \alpha_h\}$. Then

$\big[(1-\alpha_c-\hat{\tau}^{MC})+(\alpha_c-\alpha_h)S(p)\big]>0\ \forall\ p\in(0,\infty)$ as $S(p)\in(0,1)\ \forall\ p\in(0,\infty)$ and thus $\theta'(p)>0$ implies $E'(p)>0$.

We next restrict the relevant price domain of the equilibrium. Note that $D(p)\geq\hat{\tau}^{MC}$ given $\hat{\tau}^{MC}<1-\max\{\alpha_c,\alpha_h\}$. Define p_τ as the price that solves $S(p_\tau)=\hat{\tau}^{MC}$. Since $S(p)$ is strictly decreasing, p_τ is unique and $S(p)<\hat{\tau}^{MC}$ for all $p>p_\tau$. Then any solution to $S(p)=D(p)$ and thus equilibrium BGP price must satisfy $p^*\in(0,p_\tau]$ and the sufficient condition for uniqueness above reduces to $\theta'(p)>0\ \forall\ p\in(0,p_\tau]$.

Since the CES price index $P(p)$ is strictly increasing in p, it suffices to ensure monotonicity in P: $\theta'(p)\geq 0\ \forall p\in(0,p_\tau]\quad\iff\quad\theta'(P)\geq 0\ \forall P\in(P(0),P(p_\tau)]$. Given $\theta(p)>0\ \forall\ p\in(0,p_\tau]$, we can write

$$\frac{d\log\theta(p)}{d\log P(p)}=\frac{d\log\left(\frac{e_t}{x_t}\cdot\frac{p_t i_{ht}}{e_t}\right)}{d\log P(p)}=\eta_e(p)+\eta_{i_h}(p).$$

Therefore θ is weakly increasing in p whenever $\eta_e(p)+\eta_{i_h}(p)\geq 0$. Using the BGP closed-form expressions, we can derive closed-form expressions for $\eta_e(p)$ and $\eta_{i_h}(p)$.

Composition elasticity $\eta_{i_h}(p)$. Let $\theta_{i_h}(p)\equiv\frac{p_t i_{ht}}{e_t}=1-\tilde{\vartheta}(p)$, then by the chain rule we have

$$\eta_{i_h}(p)=\frac{d\log\theta_{i_h}(p)}{d\log P(p)}=\frac{d\log\theta_{i_h}(p)}{d\log p}\left(\frac{d\log P(p)}{d\log p}\right)^{-1}$$

Differentiating the CES health price index $P(p)$ from (36) yields

$$\frac{d\log P(p)}{d\log p}=\frac{\chi^\varepsilon p^{1-\varepsilon}}{(1-\chi)^\varepsilon+\chi^\varepsilon p^{1-\varepsilon}}.$$

Then $P(p)$ is strictly increasing in p. Note that $\theta_{i_h}(p)=1-\tilde{\vartheta}(p)=\frac{\chi^\varepsilon p^{1-\varepsilon}}{(1-\chi)^\varepsilon+\chi^\varepsilon p^{1-\varepsilon}}$, then log-differentiating yields

$$\frac{d\log\theta_{i_h}(p)}{d\log p}=(1-\varepsilon)\big(1-\theta_{i_h}(p)\big).$$

Then using $\theta_{i_h}(p)=\frac{d\log P(p)}{d\log p}$ and combining yields

$$\eta_{i_h}(p)=(1-\varepsilon)\frac{1-\theta_{i_h}(p)}{\theta_{i_h}(p)}.$$

Then plugging in $\frac{1-\theta_{i_h}(p)}{\theta_{i_h}(p)} = \left(\frac{1-\chi}{\chi}\right)^{\varepsilon} p^{\varepsilon-1}$ yields the final expression for the composition elasticity

$$\eta_{i_h}(p) = (1-\varepsilon)\left(\frac{1-\chi}{\chi}\right)^{\varepsilon} p^{\varepsilon-1},$$

which is negative and strictly decreasing in p for $\varepsilon > 1$, that is, if the two health goods are relatively substitutable. Note that if the two health goods are complements, $\varepsilon < 1$, then the composition elasticity is positive $\eta_{i_h}(p) > 0$ and the sufficient uniqueness condition $\eta_e(p_\tau) + \eta_{ih}(p_\tau) > 0$ is easier to satisfy, in particular, it suffices to show that $\eta_e(p_\tau) \geq 0$.

Scale elasticity $\eta_e(p)$. Let $\theta_e(p) \equiv \frac{e_t}{x_t} = (1-\vartheta_c^*(p))\,\vartheta_e^*(p)$.

$$\theta_e(p) \equiv (1-\vartheta_c^*(p))\,\vartheta_e^*(p) \quad \text{and} \quad \eta_e(p) = \frac{d\log\theta_e(p)}{d\log P(p)}.$$

Using the BGP expressions for the health spending share in savings (47a) and the young consumption share (47b), we get

$$\theta_e(p) = \frac{\Psi\,\vartheta_e^*(p)}{(1+\Psi) - \vartheta_e^*(p)},$$

where $\Psi \equiv \left(\frac{\beta}{1-\beta}R^{1-\sigma}\right)^{1/\sigma} > 0$. Define $X(p) \equiv \left(\frac{(P(p)R)^{1-\sigma}}{b\xi}\right)^{1/\sigma}$ such that the BGP health expenditure share in savings can be written as $\vartheta_e^*(p) = 1/(1+X(p))$. Then

$$\frac{X'(p)}{X(p)} = \frac{d\log X(p)}{dp} = \frac{1-\sigma}{\sigma}\frac{P'(p)}{P(p)},$$

and when $\sigma > 1$ and $P'(p) > 0$, we have $X'(p) < 0$ and therefore $\vartheta_e^*(p)$ is strictly increasing in p. By the chain rule we have

$$\eta_e(p) = \frac{d\log\theta_e(p)}{d\log P(p)} = \frac{1}{\theta_e(p)}\frac{\partial\theta_e}{\partial\vartheta_e^*}\frac{d\vartheta_e^*(p)}{d\log P(p)},$$

where

$$\frac{\partial\theta_e}{\partial\vartheta_e^*} = \frac{\Psi(1+\Psi)}{\left((1+\Psi) - \vartheta_e^*(p)\right)^2}, \qquad \Rightarrow \qquad \frac{1}{\theta_e}\frac{\partial\theta_e}{\partial\vartheta_e^*} = \frac{1+\Psi}{(1+\Psi)-\vartheta_e^*(p)} \cdot \frac{1}{\vartheta_e^*(p)}.$$

Next, write $\vartheta_e^*(p) = (1 + X(p))^{-1}$ with $X(p) \equiv \left(\frac{(P(p)R)^{1-\sigma}}{b\xi}\right)^{1/\sigma}$. Then

$$\frac{dX(p)}{d\log P(p)} = X(p)\frac{1-\sigma}{\sigma},$$

and thus

$$\frac{d\vartheta_e^*(p)}{d\log P(p)} = -\frac{1}{(1+X(p))^2}\frac{dX(p)}{d\log P(p)} = \vartheta_e^*(p)\big(1-\vartheta_e^*(p)\big)\frac{\sigma-1}{\sigma}.$$

where we used $X = \frac{1-\vartheta_e^*}{\vartheta_e^*}$ in the last step. Then combining yields

$$\eta_e(p) = \frac{\sigma-1}{\sigma}\frac{(1+\Psi)\big(1-\vartheta_e^*(p)\big)}{(1+\Psi)-\vartheta_e^*(p)}.$$

Finally, since $\vartheta_e^*(p)$ is increasing in p when $\sigma > 1$, it follows that $\eta_e(p)$ is strictly decreasing in $p > 0$. Since $\eta_e(p)$ is decreasing and $\eta_{i_h}(p)$ is decreasing on $(0, \infty)$ for $\varepsilon > 1$, the overall private health demand elasticity $\eta_e(p)+\eta_{i_h}(p)$ is decreasing in p for $\varepsilon > 1$. Thus for $\varepsilon > 1$ $\eta_e(p_\tau)+\eta_{i_h}(p_\tau) \geq 0$ implies $\eta_e(p)+\eta_{i_h}(p) \geq 0$ for all $p \in (0, p_\tau]$ and $\eta_e(p_\tau)+\eta_{i_h}(p_\tau) \geq 0$ is a sufficient condition for uniqueness of the interior balanced growth path. If $\varepsilon < 1$, then $\eta_{i_h}(p) > 0$ for all $p > 0$, and $\eta_e(p) \geq 0$ for all p given $\sigma > 1$ by its closed-form expression above. Thus $\eta_e(p) + \eta_{i_h}(p) > 0$ for all $p \in (0, p_\tau]$.

For the Cobb-Douglas case $\varepsilon = 1$, $\theta_{ih}(p) = 1 - \tilde{\vartheta}(p) = \chi$ is constant, so $\eta_{ih}(p) = 0$. Since

$$P(p) = \frac{p^\chi}{\chi^\chi(1-\chi)^{1-\chi}}$$

is strictly increasing in p, and $\eta_e(p) > 0$ for all p when $\sigma > 1$ by the expression above, $\theta(p) = \theta_e(p)\theta_{ih}(p)$ is strictly increasing on $(0, p_\tau]$. Hence $\theta'(p) > 0$ on the relevant price domain and, under condition (56), the same argument implies $E'(p) > 0$. This establishes uniqueness also for $\varepsilon = 1$. Equivalently, condition (57) holds automatically since $\eta_e(p_\tau) + \eta_{ih}(p_\tau) = \eta_e(p_\tau) > 0$. □

C Calibration Appendix

C.1 Health Investment Production

The core of the model on the household side consists of the two functional forms that map households' health investment choices, i_{ct} and i_{ht}, into their survival rate and, thus, life expectancy,

$$\begin{aligned} i_t &= \left[\chi\left(i_{ht}+\nu\right)^{\rho}+(1-\chi)\left(i_{ct}+\nu\right)^{\rho}\right]^{\frac{1}{\rho}} \\ \psi(i_t) &= 1-\left[1+i_t\right]^{-\xi}. \end{aligned}$$

The mapping is thus parametrized by (ξ, ν, χ, ρ) which we calibrate internally. We calibrate ξ targeting an (absolute) elasticity of the mortality rate with respect to modern health spending for a 60-year-old individual of 0.16 from Hall and Jones (2007). They estimate the absolute elasticity of nonaccident mortality with respect to age-specific medical spending (reported in Figure 3 of the paper) using data between 1950 and 2000 as

$$\left|\frac{\%\Delta\pi(1950 \to 2000)}{\%\Delta(p_h i_h)(1950 \to 2000)}\right|. \tag{C.1}$$

We compute the elasticity in the model the same way for the 1940-1980 and 1980-2020 periods and target the average of the two. We calibrate the non-homotheticity parameter ν to match initial life expectancy in phase 1, which, given ξ, maps directly to life expectancy in the absence of active health investment, $i_{ct} = i_{ht} = 0$ given by

$$\psi_{\text{phase 1}} \equiv \psi(i_t = \nu) = 1-(1+\nu)^{-\xi} \quad \Longrightarrow \quad \text{LE}_{\text{phase 1}} = (1+\psi_{\text{phase 1}}) \cdot 40.$$

Thus we set ν to exactly match our estimate of remaining cohort life expectancy at age 20 in year 1820 (the beginning of our sample) given by

$$\nu = \left(\frac{40}{80-\text{LE}_{\text{phase 1}}}\right)^{\frac{1}{\xi}} - 1, \tag{C.2}$$

where $\text{LE}_{\text{phase 1}} = \text{LE}_{1820} = 40.44$ years. We internally calibrate the relative shares of modern and basic health goods in the health investment function, χ, to match life expectancy at the first kickoff. Given that the modern health sector is inactive in 1860, which we ensure as outlined in Section C.5, we calibrate χ such that the threshold level of cash at hand at which households begin to demand basic health goods satisfies $x_{1820} < \underline{x}_1(\underline{p}) < x_{1860}$ to match the timing of the first kickoff in 1860. Within these bounds, we select χ targeting life expectancy in 1860, which is fully determined by the level of basic health spending, i_{c1860}.

Lastly, we discipline the substitution elasticity between basic and modern health goods, governed by ρ, from the time-series of the modern health spending share because ρ governs how households' expenditures on health shift from basic to modern health goods as its relative price, p_t, changes over time, thereby determining the speed of structural change towards the modern health sector. Specifically, we calibrate ρ to match the rise in the spending share of modern health goods in GDP, $\frac{p_h y_h}{y}$, between 1940 and 1980.[42]

C.2 Preferences

We externally set the weight on second period utility relative to first period utility, β, to match an annualized discount factor of 0.96. We discipline the base value-of-life parameter, b, through the value of a statistical life. Empirical estimates of the value of a statistical life are dispersed, ranging from about two million dollars to nine million dollars or more, as discussed by Viscusi and Aldy (2003). We follow Hall and Jones (2007) and use the value of three million dollars from the U.S. Department of Transportation, which sits in the range of empirical estimates. Specifically, we internally calibrate b targeting a model-implied willingness to pay when young for a marginal increase in survival to old age, defined as

$$VSL_t^y = \frac{\beta}{1-\beta}\frac{u(c_{t+1}^o)}{u'(c_t^y)},$$

of \$3 million in 2000. We construct the year-2000 model value by linear interpolation between 1980 and 2020. To express the model-implied object in year-2000 dollars, we first compute VSL_t^y as a share of GDP per capita and then multiply by U.S. real GDP per capita in year-2000 dollars. Lastly, we set the inverse of the intertemporal elasticity of substitution (IES) to ensure the existence of an interior balanced growth path, $\sigma = 1+\xi$.

[42] In Section C.7, we show that the evolution of the model-implied modern health spending share over time varies significantly with ρ, making this share an informative moment to discipline ρ. Similarly, we show that the *cross-sectional* income-life expectancy gradient is not very informative about ρ, in the sense that the model-implied sensitivity of life expectancy to income changes in the cross-section, holding the price of modern health goods constant, does not vary much with ρ. The key distinction between the cross-section and the time series is that in the latter not only income but also the price of modern health goods is changing (endogenously) as well.

C.3 Technology

We externally set the 40-year gross real return factor to 3.3, which corresponds to an annual net real interest rate of 3%. We externally set the intermediate good intensities in the two sectors, which translate into the capital shares in the aggregate production function, to $\alpha_c = 0.33$ and $\alpha_h = 0.22$, the former consistent with standard estimates for the US, and the latter based on estimates from Acemoglu and Guerrieri (2008). Lastly, we set the elasticity of the innovation probability with respect to R&D spending, $\gamma = 0.35$, based on estimates of the elasticity of patents with respect to R&D spending from Hausman, Hall, and Griliches (1984).[43] We internally calibrate the step size of innovations in the R&D process in both sectors, λ, targeting average GDP per capita growth between 1820 and 2020 of approximately 1.5%, based on Figure 1.

C.4 Government

Public health spending. Given the government budget constraint, public health spending on the old in the model is equal to the corresponding tax levied on the young. Thus we calibrate the tax as a share of GDP, $\hat{\tau}_t^{MC}$ to match public health spending on the old in the data. We set public health spending on the old to zero in all model periods prior to the introduction of the US Medicare system in 1965, $\hat{\tau}_{t<1980}^{MC} = 0$. In 1980 and 2020, we calibrate the tax to satisfy $\hat{\tau}_t^{MC} = CBO_t$, where CBO_t is the sum of Medicare spending and the nursing home component of Medicaid as a share of GDP from the CBO and CMS.[44] From 2020 onward, we assume that nominal per-old-capita public health spending as a share of GDP stays constant at its 2020 level and satisfies

$$\frac{i_{ht}^{MC} p_t}{y_t} = \frac{i_{h2020}^{MC} p_{2020}}{y_{2020}} \; \forall \; t \geq 2020, \tag{C.3}$$

where i_{ht}^{MC} is real per-old-capita public health spending. With constant nominal per-old-capita spending in GDP, the size of overall public health spending then grows endogenously at the same rate as the size of the old-age population, n_t^o, and converges to a constant share of GDP over time as n_t^o converges to 1 on the BGP, given by $\hat{\tau}^{*MC} = \hat{\tau}_{2020}^{MC} / n_{2020}^o$.

[43] Consistent with a broader literature, Hausman et al. (1984) find elasticities in the range of 0.3-0.6 across several specifications. We choose their fixed-effect estimate of 0.35 (Table 2, col. 5) as it most closely corresponds to the within-firm, contemporaneous elasticity that we want to capture.

[44] Medicare is from the CBO's Historical Budget Data workbook, sheet "5a. Mandatory Outlays (GDP)". The Medicaid nursing-home component is from CMS Historical National Health Expenditure Accounts, table "National Health Expenditures by type of service and source of funds, CY 1960–2024," category "Nursing Care Facilities and Continuing Care Retirement Communities."

We allow public health spending to increase utility from being alive in old-age for generation t as follows:

$$b_t = b + b_t^{MC}, \tag{C.4}$$

where b is the time-invariant base value of life and b_t^{MC} is the endogenous cohort-specific value derived from old-age health spending. We calibrate b_t^{MC} for each generation such that the representative worker of that cohort is indifferent to the overall program, that is, the period-t young household is indifferent between participating in the program, in which case she pays the tax τ_t^{MC} and enjoys additional quality of life b_t^{MC} upon reaching old-age, and opting out of the program, in which case she does not pay the tax and only enjoys base value of life b in old-age. Formally, given the equilibrium level of cash at hand net of taxes, x_t^{MC}, we solve the following condition for lifetime utility of generation t

$$U_t(x_t^{MC}, p_t;\ b + b_t^{MC}) = U_t(x_t^{MC} + \tau_t^{MC}, p_t;\ b), \tag{C.5}$$

This assumption makes the direct private effect of Medicare and Medicaid utility-neutral cohort by cohort and isolates the general-equilibrium effects of public old-age health spending operating through demand.[45]

We set the public R&D subsidy on health research in 1940 to $s_{h,1940} = 0.2$ to match the size of the health-related R&D program (CMR) relative to private biomedical research spending prior to the program in 1940.[46]

C.5 Initial Conditions.

The model has three state variables $\Phi_t = (q_{ct-1}, q_{ht-1}, T_t)$ until 2020. After 2020, the current old population, n_t^o, becomes an additional state variable to determine public health spending through Medicare and Medicaid. We keep track of previous sectoral qualities, (q_{ct-1}, q_{ht-1}), to compute current sectoral qualities according to the law of motion for the aggregate quality index (21), and transfers from the deceased, T_t, are required to

[45] Since $\sigma > 1$, the consumption term $c^{1-\sigma}/(1-\sigma)$ vanishes along the asymptotic BGP, and the cohort-specific shifter b_t^{MC} pinned down by the indifference condition above converges to zero. Accordingly, the long-run characterization in Section 4.3 and Section B.3 is unchanged and is written with the constant value b.

[46] The CMR budget was about $25 million between 1941 and 1945, or $5 million per year on average, see Gross and Sampat (2025) and Ladimer (1954). Industry biomedical research spending was around $25 million in 1940 according to Bloom and Randolph (1990).

compute current cash at hand, x_t, of the young. The first period of the model is 1820 and thus we choose initial conditions $\Phi_1 = \Phi_{1820}$.

We initialize quality in the consumption goods sector, q_{c0}, such that the initial income level of the economy, x_1, makes households exactly indifferent to surviving into the second period (no-suicide condition) pinned down by

$$0 = \beta\psi_0 u(Rs_1),$$

We calibrate the initial quality in the health goods sector, q_{h0}, to match the timing of the modern health sector take-off and in particular its output share in 1940. We initialize the health sector sufficiently unproductive such that it is inactive prior to 1940 and first becomes operative in 1940. Conditional on matching the kickoff timing, we then pin down q_{h0} to match the health output share in 1940 in the data (see Section 2).[47] Transfers in period 1 are determined by savings and the mortality rate of the previous period, $T_1 = Rs_0(1 - n_1^o)$. We initialize initial savings and thus transfers such that the small open economy has a zero net foreign asset position in period 1 so that $k_1 = s_0$.

Existence and Uniqueness of the BGP. Given the quantitative calibration, we evaluate the sufficient conditions for existence and uniqueness of the interior asymptotic BGP derived in Proposition 7. The BGP exists since $\sigma = 1 + \xi > 1$. Given $\hat{\tau}^{*MC} = 0.088$, $\eta_e(p_\tau) = 0.17$ and $\eta_{i_h}(p_\tau) = -0.06$, evaluating (56) and (57) yields uniqueness.

C.6 Income and life expectancy in the cross-section

Our theory interprets the empirical evidence on income and life expectancy presented in Section 2 as rising household demand for life expectancy (LE) driven by rising income over time. Therefore, we calibrate the mechanism that leads from income to life expectancy in the time-series for a representative cohort. To further validate the resulting strength of this mechanism, we next compute the model-implied income-LE gradient in the cross-section and contrast it with empirical estimates reported by Chetty et al. (2016).

While the benchmark model features representative cohorts without income heterogeneity, we find it useful to construct the model's cross-sectional life expectancy distribution for different income levels in partial equilibrium for two reasons. First, we want

[47] Recall from (43) that there is a negative relationship between the health quality and minimum health price consistent with labor market clearing. Thus, we calibrate the initial health quality q_{h0} to ensure that the health price is too high for households to demand modern health goods prior to 1940, given Proposition 3, and sufficiently low for the health sector to emerge in 1940.

to assess whether the implied cross-sectional income-LE gradient is empirically plausible. Second, in order to additionally validate our calibration strategy of disciplining the substitution elasticity ρ with the empirical time-series of modern health spending, we show that the cross-sectional income-LE gradient, quantitatively, is not informative about ρ.

For the 1980 and 2020 cohort of the model, we compute life expectancy for different income levels[48], keeping the price constant across income levels at its respective equilibrium value, p_{1980} and p_{2020}. Specifically, we do the following:

1. **Income grid relative to median.** We construct the distribution of income relative to its median from the 1980 and 2020 Census income distribution (household dollar income) as a grid $r = \{r_{Q2}, 1, r_{Q4}\}$, where $r_{Qj} \equiv$ (mean income in quintile Qj)/(mean income in quintile Q3). We use Q3 as a proxy for the median.

2. **Cohort-t LE along the grid.** For the cohorts $t = 1980$ and $t = 2020$ (e.g., the cohort aged 20 in t), we re-solve the young household problem at the fixed period-t equilibrium price, p_t, across the income grid, $x_t \cdot r$, where x_t is the period-t equilibrium level of cash at hand, which we take to be the mean income in the third quintile and median income, and r is the grid defined in the previous step. Given the household choices along the cross-sectional income grid, we then construct period-t cross-sectional life expectancy as

$$\mathrm{LE}_t(x_Q) = \{\mathrm{LE}_t(x_{Q2}), \mathrm{LE}_t(x_{Q3}), \mathrm{LE}_t(x_{Q4})\}.$$

C.7 Calibrating the substitution elasticity between health goods

We next discuss our calibration strategy of the elasticity of substitution between basic and modern health goods as it is a key parameter driving the transition. In particular, we show that in the model the *time series* of modern health spending is highly sensitive to ρ, while the *cross-sectional* income-LE gradient is not, validating our calibration choice of disciplining ρ based on the time series of modern health spending.

First, to assess the importance of ρ for the time-series of modern health spending, we re-solve the benchmark from 1940 onward for different values of ρ, where we keep all other parameters at the benchmark calibration, and we also keep the state variables

[48] We use the 2022 Current Population Reports by Guzman and Kollar (2023), Table A-4a, "Selected Measures of Household Income Dispersion: 1967 to 2022" to construct the income grid employed in the model.

coming into 1940 from the benchmark economy, thus, changing ρ only from 1940 onward (the year when the health sector becomes active in the model).

Figure 7: Modern health spending in GDP for different substitution elasticities.

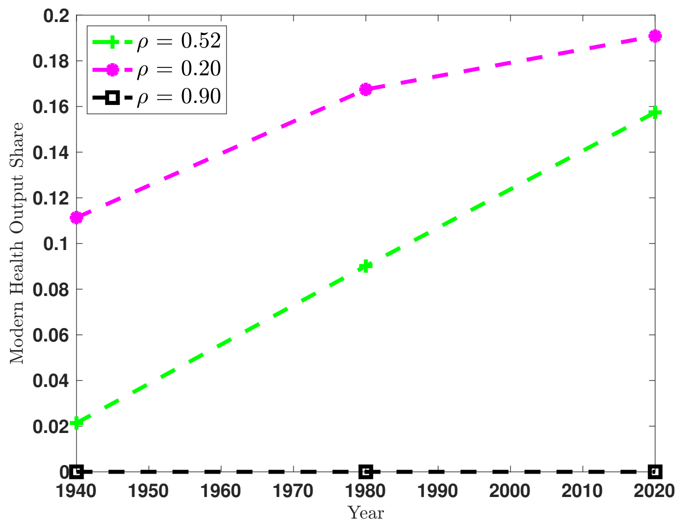

Figure 7 plots the modern health spending share in GDP across different substitution elasticities. As this elasticity falls, the ability of households to substitute basic health for modern health goods falls, forcing them to purchase more of the expensive modern health good to reach the same level of longevity. As a result, the spending share on modern health rises significantly relative to the benchmark economy. Similarly, raising the substitution elasticity allows households to more effectively substitute cheap basic health goods for modern health goods, reducing spending on modern health. As a result, the modern health sector does not emerge at all by 2020.

Second, to assess the importance of ρ for the life expectancy - income gradient in the cross-section, we recompute the partial equilibrium cross-sectional life expectancy distribution as outlined above with different values for the substitution elasticity, $\mathrm{LE}_t(x_Q; \rho)$, where we leave all other parameters as in the benchmark calibration, and we take the equilibrium price, p_t, and median income, x_t, also from the benchmark economy with the calibrated $\rho = 0.52$.

Table 9: Model-implied cross-sectional income-LE gradient for the 2020 cohort for different substitution elasticities.

ρ	LE increase per 5 pct
0.20	0.57
0.52	0.55
0.90	0.46

Table 9 shows that the cross-sectional income-LE gradient, expressed as the years of life expectancy gains for every 5-percentile increase in the income distribution, varies only slightly with the elasticity of substitution, suggesting that ρ in the model is not primarily informative about this moment.