The Demographic Cliff and the Market for Higher Education: Implications for Public Finance*

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Abstract

We construct a general equilibrium life-cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. The cross-sectional distribution of college quality and major specialization is an equilibrium outcome and is shaped by the demographic structure of the economy, by public education spending and college loan policies, as well as by the endogenous relative wages that college graduates with different majors command in the labor market. We use the model to evaluate the aggregate and distributional consequences of the "demographic cliff", a scenario with lower population growth that will reduce the number of high-school graduates in the coming decades. We then discuss how public education finance reforms by the current administration shape these outcomes.

JEL Classification: D15, D31, E24, I24

Keywords: Equilibrium Model of College Market, Education Subsidies, Demographic Cliff, Technological Change, College Major Choice.

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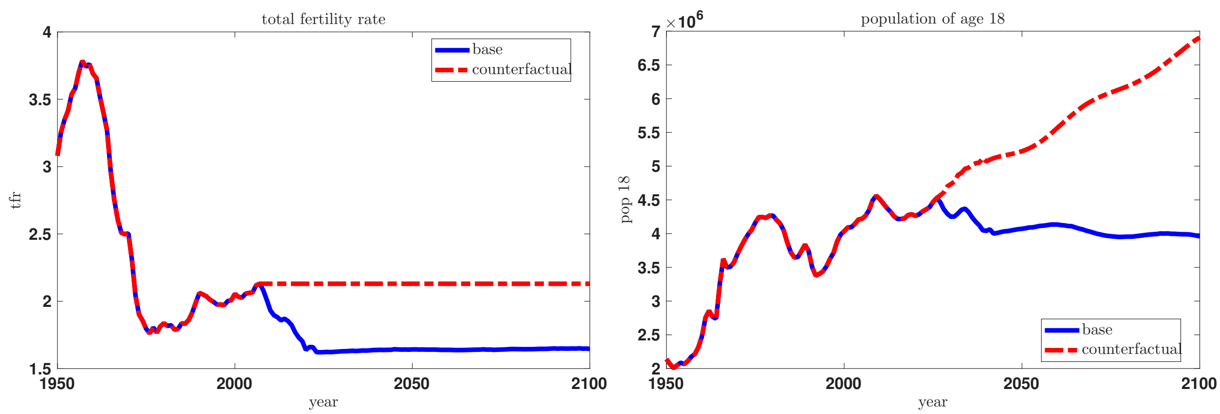
1 Introduction

The U.S. is undergoing a demographic cliff that is expected to have a profound impact on higher education and the labor market in the coming decades. Just between 2025 and 2032 the number of domestic high school graduates is expected to fall by 5% (NCES Digest of Education Statistics, 2023). Figure 1 shows the actual and predicted fertility rate in the US and illustrates the number of potential college students lost due to the decline of the fertility rate since 2008.

Figure 1: Fertility Rate and Population Size



(a) Total Fertility Rate (b) Population of Age 18

Notes: Projected fertility rate and population of age 18. Own calculations based on UN and HMD.

The college supply side is already responding to this demographic shift. About 100 institutions closed between AY 2022/23 and AY 2023/24, according to 2024 NCES statistics, and 2020 enrollments were ca 10% lower than in 2010, according to the 2023 NCES Digest of Education Statistics. Not only will lower enrollments eventually translate into smaller graduating classes, but there are concerns that shortages in specific areas such as STEM fields, will be especially prominent. BLS (2025) estimates that STEM-based occupations are expected to grow faster than non-STEM occupations. At the same time, STEM fields (both in college and in the labor market) are populated heavily by immigrants, see e.g., (NSF Board, 2024) and changes in immigration policy will (and perhaps already have) exacerbate domestic concerns about skill shortages in key labor market areas. Furthermore, there is evidence (see, e.g., Deming and Noray, 2020) that STEM careers are getting shorter and are subject to increased risk of technological obsolescence.

The broad question we are concerned with is how private education choices and public education policy should respond to these demographic and technological trends, and how do these trends alter the trade-offs involved in designing public education and redistributive policies? Specifically, we construct a general equilibrium life cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. The cross-sectional distribution of college quality and college specialization is an equilibrium outcome and is shaped by the demographic structure of the economy, by public education spending and college loan policies, as well as by the endogenous relative wages that college graduates with different majors command in the labor market. We use the model to evaluate the aggregate and distributional consequences of the "demographic cliff" that will reduce the number of high-school graduates in the next decades, and against the backdrop of technological change (e.g., the emergence of AI) in the labor market that changes the relative demand for STEM-intensive occupations and college majors. We then quantify the aggregate and distributional effects of recent federal funding cuts in higher education, focusing on their impact on enrollment, the composition of college quality, tuition levels, wages, and public finances.

To conduct our analysis we embed a life cycle model with college quality and college major choice into the competitive equilibrium model of the college market in the spirit of Cai and Heathcote (2022). Households that differ in their initial resources (due to, e.g., inter-vivos transfers by their parents), their innate ability to study and their residence status (determining whether they qualify for in-state tuition at public colleges) first decide whether and what quality colleges they want to attend, given the equilibrium tuition schedule that depends on these characteristics. After enrolling in college, they are subject to college-major specific preference shocks, and given these shocks as well as their expected labor earnings processes from a given college major they decide which major to pursue. After college, household life follows a standard life cycle with consumption-savings decisions, eventual exogenous retirement and stochastic death. On the college supply side, competitive colleges can freely enter the market at each level of college quality, where quality is determined by a combination of average student quality as well as per-student discretionary spending. The college-quality-, student-ability-, and residence-status-specific tuition schedule is determined in competitive equilibrium by household demand and the supply of competitive colleges. The key mechanism of the college market model operates through the ability composition of the student body. Changes in enrollment alter the share of high-ability students at each college type, which in turn affects instructional spending and the equilibrium tuition discounts offered to attract high-ability students. These pricing adjustments then feed back into households' college and major choices.

After calibrating the model to U.S. macroeconomic, demographic, and education data, we analyze three counterfactual experiments: (i) the removal of public subsidies to higher education, (ii) a demographic transition characterized by declining population growth, and (iii) the joint impact of these two shocks. We first use the model to characterize how, according to the model, the demographic cliff, modeled as a permanent decline in the U.S. fertility rate starting in 2008 impacts the U.S. college market, the labor market and public finances in the long run. In order to quantify the importance of these demographic shifts we then consider a counterfactual (from the perspective of today) demographic scenario in which the fertility rate is at replacement level and therefore the population is constant in the long run. Finally, for both demographic scenarios we evaluate the aggregate, distributional, college market, public finance and welfare consequences of abolishing public subsidies for higher education.

The declining population due to the demographic cliff, perhaps not surprisingly, has large negative effects on macroeconomic aggregates in the long run, simply because the population shrinks by 30.6%, relative to the counterfactual scenario without changes in demographics. However, per-capita outcomes are less detrimental. First, the demographic cliff has modest but favorable effects on the college market. Concretely, the long-run college enrollment rate increases and the distribution of students shifts toward middle-quality colleges. However, changes in the ability composition of individuals deciding to go to college reduce the share of STEM majors in the middle segment of the quality distribution. Overall, on account of dwindling absolute numbers of high-school graduates, colleges experience a decline in absolute student numbers throughout the quality distribution. The share of high-ability students remains almost unchanged at top-quality colleges but falls substantially at low- and medium-quality colleges due to the inflow of less-prepared marginal students, which in turn increases the tuition-ability discount in the lower and middle segments of the quality distribution.

Given our overall focus on the interaction between public education policies and long-run demographic and technological trends (or shocks), we next evaluate a public college funding cut akin to the one recently implemented by the current administration, both under a baseline scenario with the initial steady-state population growth rate and under the demographic cliff scenario described above. We find that the long-run *aggregate* losses induced by a public funding cut are much smaller than those generated by the demographic shock. However, the subsidy cut qualitatively undoes the positive effects of the demographic shock on college enrollment, the quality distribution of students, and college tuition. When the subsidy cut takes place against the backdrop of the demographic cliff, perhaps the currently most relevant *factual* scenario, the joint impact of the two shocks on public finances exceeds the sum of their individual effects, implying that demographic change amplifies the fiscal consequences of higher education funding cuts. The amplification of the fiscal strain happens because the demographic shock creates a fragile, enrollment-driven fiscal buffer that the subsidy cut dismantles - confronting the economy with both a smaller workforce and a weaker skill composition simultaneously.

1.1 Related Literature

This paper builds on and contributes to four strands of literature: (i) the consequences of demographic change for higher education demand and the college landscape; (ii) equilibrium models of the college market with heterogeneous institutions, endogenous tuition, and student sorting; (iii) technological change, skill demand, and the returns to college majors; and (iv) macro public finance, education policy, and long-run fiscal sustainability.

Demographic change, higher education demand, and the changing college landscape. A large literature studies the implications of demographic change for education demand, college enrollment patterns, and institutional stress in higher education. This literature documents the decline in the number of college-aged individuals, the associated pressure on college enrollments, and the risk of institutional closures or financial distress (National Center for Education Statistics, 2023; Kelchen et al., 2024; Conesa et al., 2020). More broadly, it highlights that long-run demographic trends are likely to affect not only aggregate labor supply but also the educational composition of the workforce. Empirical work has also documented how demographic changes affect education demand and skill composition across countries and regions, with implications for labor supply and productivity, e.g. Hendricks and Schoellman (2023). In the quantitative macroeconomics literature, Ludwig et al. (2012) show that endogenous human capital accumulation is an important adjustment channel dampening the welfare consequences of demographic aging in a general equilibrium OLG model. This paper builds on this literature by moving from descriptive evidence on enrollment decline, college financial hardship, and college closures to an equilibrium analysis of how colleges adjust tuition schedules, educational quality, and program offerings in response to demographic change.

Equilibrium models of the college market and the supply side of higher education. A second strand studies higher education as an equilibrium market with heterogeneous institutions, endogenous tuition, admissions, student sorting, and financial aid. Important contributions in macroeconomics and industrial organization include Cai and Heathcote (2022), Alon et al. (2025), Capelle and Matsuda (2025), Arcidiacono (2005), Epple et al. (2006), Fu (2014), and Marto and Wittmann (2024). Related work also emphasizes quality differences across colleges, mismatch, and the consequences of differential college access for inequality and intergenerational mobility (Blair and Smetters, 2021; Hendricks et al., 2021,?, 2024). This paper builds directly on this literature, but embeds the college market into a broader macro–public finance environment in which demographic transition, technological change, early-life human capital accumulation, and government policy jointly shape the equilibrium structure of higher education.

Technological change, skill demand, and the composition of human capital. Another related literature studies how technological change affects the labor market returns to education, occupational structure, and the demand for specific skills. This includes work on skill-biased technological change and capital–skill complementarity (Krusell et al., 2000; Caunedo et al., 2023), as well as more recent evidence on job displacement, skill obsolescence, and changing returns to college majors and occupations (Deming and Noray, 2020; Braxton and Taska, 2023; Hendricks and Schoellman, 2023; Kogan et al., 2023). Closely related empirical work studies major choice, the returns to different fields of study, and the changing task content of jobs (Altonji et al., 2016; Altonji and Zimmerman, 2018; Altonji et al., 2012, 2014). Cross-country empirical work also highlights the role of human capital in technology adoption and diffusion (Sunde and Vischer, 2015; Cervellati et al., 2023). This paper contributes to this literature by providing a quantitative framework in which the returns to different college majors, the equilibrium composition of human capital across fields of study, and the response of the college market to technological and demographic shocks are jointly determined in general equilibrium.

Macro public finance, education policy, and long-run fiscal consequences. This paper is also closely connected to a literature in macro public finance that studies how taxes, transfers, education subsidies, public insurance, and borrowing constraints shape human capital accumulation, inequality, and long-run fiscal outcomes. Educational choices affect not only private earnings and mobility, but also productivity of future cohorts, thus the future tax base and the sustainability of public finances. In particular, this paper builds on recent work on education finance reform and the distributional consequences of education policy (Krueger et al., 2025). Its contribution is to analyze these public-finance questions in a unified dynamic setting in which the structure of higher education is endogenous and policy reforms can be evaluated along both transition paths and long-run equilibria.

An important overall contribution of this paper is to build an integrated quantitative framework in which education and human capital investment decisions, the college market, labor-market outcomes, and public policy interact jointly over demographic and technological transitions.[1] This allows us to study not only aggregate enrollment and average skill levels,

[1] Modeling intergenerational linkages explicitly — with endogenous parental investment in the human capital of their children and intergenerational wealth transfers, in the spirit of our own earlier work (Krueger et al., 2025) — would be an interesting extension of the current framework. We abstract from intergenerational extensions and endogenous pre-college human capital in the current paper.

but also the composition of human capital, the allocation of talent across institutions and fields of study, and the implications of these adjustments for inequality, intergenerational mobility, and fiscal sustainability.

2 The Model

Time is indexed by t and in each period a new cohort of size $N_{0,t}$ is born. The size of newborn cohorts $N_{0,t}$ is exogenous but may vary over time. There is a unit mass of households in each cohort. Households live from age $j = 0$ (which represents the college phase or the first labor market period of life) to at most age $j = J$, so that life after college starts at age $j = 1$. The retirement age j_r is exogenous and after this age households face age-specific survival risk: the probability of surviving from age $j-1$ to age j is given by ψ_{j-1}, with $\psi_j = 1$ for $j \leq j_r - 1$ and $\psi_J = 0$. Therefore, the number of individuals of age j alive in period t is defined recursively as

$$N_{t+1,j+1} = \psi_j N_{t,j} \tag{1}$$

We will describe the model in a stationary recursive setting, therefore suppressing time subscripts t until we introduce unexpected MIT transitions. The stationary population structure is then denoted by the vector $(N_0, ...N_J)$ and determined by (1) as well as the normalization $N_0 = 1$.

2.1 Household Endowments, Preferences, Budget Sets and Decision Problems

2.1.1 Endowments and Budget Constraints

Households are endowed with innate ability e which falls into a discrete set $e \in \mathcal{E} = \{e_1, e_2, \ldots, e_E\}$, and we denote by $\mu_e(.)$ the measure of households of a given e. Following Cai and Heathcote (2022), we assume that in addition to ability households also differ by residence status $r \in \mathcal{R} = \{r^i, r^o\}$ which determines the generosity of government subsidies received by universities when these households become students (and hence the tuition these students pay).We denote by $\mu_r(.)$ the measure of households in residence r. In addition, households start with initial resources $b \in \mathcal{B}$, interpreted as initial wealth transfers from parents, which are drawn from an exogenous child ability-specific probability measure $F_e(.)$. Thus, initial household heterogeneity is summarized by the vector $(e, r, b) \in \mathcal{E} \times \mathcal{R} \times \mathcal{B} \equiv \mathcal{S}$. Probability measures over cohort age j populations will be denoted generically by Φ_j and are endogenous for ages $j \geq 1$, but the measure Φ_0 is exogenous and determined by the initial distributions (μ_e, μ_r, F_e) for the economically newborn (aged 18 in the real wold) cohort.[2]

Households can choose to attend colleges of different varieties, where a given college type is characterized by a quality $q \in \mathcal{Q} = \{0, [q_{\min}, q_{\max}]\}$, (and $q = 0$ is non-college and $q_{\min}$ is the lowest and $q_{\max}$ is the maximum college quality available on the market), as well as a program $p \in \mathcal{P}$ or specialization. We think of $\mathcal{P}$ of a small finite and fixed set, which in our application consists of three elements ($p = 0$, no-college, $p = 1$, non-STEM college and $p = 2$, college with STEM specialization). In contrast, a college of any quality in the interval $[q_{\min}, q_{\max}]$ can endogenously emerge in equilibrium. Households choose a college type $(q, p) \in \mathcal{Q} \times \mathcal{P}$. College students are subject to exogenous college completion risk: with probability $\pi_d(q)$ they do not complete college. Since we assume that dropouts are liable for the full tuition, dropout risk only affects household choices, but has no impact on the college supply side.

College Age In the first period of life, in addition to the wealth transfer by their parents b, households earn an exogenous income y_0, standing in for working full-time at the non-college wage. If they attend college, which we denote by indicator $\mathbb{1}_{q>0}$, households lose a fraction ω of that income, which represents the opportunity cost of time required to study (and which is the purpose of introducing y_0 in the first place). Gross income is therefore $(1 - \mathbb{1}_{q>0}\omega)\, y_0$ on which households pay taxes according to a possibly non-linear tax function $T(\cdot)$. Net labor income is $y_0^n(q) = y_0 (1 - \mathbb{1}_{q>0}\omega) - T(y_0 (1 - \mathbb{1}_{q>0}\omega))$. Besides labor income taxation, households are also subject to proportional capital income and consumption taxes, with rates τ_k and τ_c, respectively.

If a household attends college ($\mathbb{1}_{q>0}$), she receives a potentially means tested transfer that is a fraction $\varsigma(b)$ of the ability e, residence r and quality q dependent college tuition $t(e, r, b; q, p)$ which is determined in equilibrium. Denoting by c_0 consumption and by a_1 asset choices of households in the first period of life, the budget set for the first period of life is defined by

$$(1 + \tau_c)c_0 + a_1 = b + y_0 (1 - \mathbb{1}_{q>0}\omega) - T(y_0 (1 - \mathbb{1}_{q>0}\omega)) - \mathbb{1}_{q>0}(1 - \varsigma(b))t(e, r, b; q, p) \tag{2}$$

$$a \geq -\underline{a}_1(q) \tag{3}$$

[2]Concretely, denote by $\mathbf{B}(\mathcal{E})$, $\mathbf{B}(\mathcal{R})$, and $\mathbf{B}(\mathcal{B})$ the (Borel) σ-algebras of $(\mathcal{E}, \mathcal{R}, \mathcal{B})$ and by $\mathbf{B}(\mathcal{S})$ the σ-algebra of $\mathcal{S}$. Then for any $E \subset \mathbf{B}(\mathcal{E})$, $R \subset \mathbf{B}(\mathcal{R})$ and $B \subset \mathbf{B}(\mathcal{B})$ we have $\Phi_0(E, R, B) = \mu_e(E)\mu_r(R) \sum_{e \in E} F_e(B)$.

where $\underline{a}_1(q)$ denotes the education-specific borrowing limit: only households that choose $q > 0$ are allowed to borrow for their college education (at a level we specify in the calibration section), whereas for those with $q = 0$ we impose a tight borrowing constraint at zero on the asset choice a_1. The direct utility of going to college is given by

$$\mathbb{1}_{q>0} \left(\nu_0 + \nu_1 \ln(q) + \mathbb{1}_{p=2} \nu_2(e) \right) \tag{4}$$

where ν_0 captures the utility flow from attending any college, $\nu_1 \ln(q)$ encodes the direct preference for high-quality college and $\nu_2(e)$ measures the (dis-)utility for studying STEM, which is assumed to be student e ability-dependent.

Working Life Ages After college, for all ages $j \in \{1, \ldots, j_r\}$ households participate in the labor market; if they have chosen $q = 0$, their labor market career commences at age $j = 0$ already. Their labor income is (potentially) stochastic and depends on the quality of a college attended as well as the program studied. The deterministic age-productivity component depends only on whether a worker attended college. Denote by $s \equiv \mathbf{1}\{q > 0\} \in \{0, 1\}$ the college attendance indicator, and let $\gamma_j(s)$ be the base age-productivity profile common to all workers with schooling status s. For college graduates, productivity additionally depends on college quality q and program choice p.

Motivated by Mincer-type earnings regressions, we assume that log productivity depends linearly on a transformed measure of college quality and major. In particular, we map college quality into an index $f(q)$ that is increasing in q but exhibits diminishing returns, reflecting the idea that the marginal productivity gains from attending increasingly higher-quality colleges eventually flatten out. Productivity for college graduates is then given by

$$\gamma_j(q, p) = \exp(\gamma_j(s) + f(q) \left(\beta_1 + \beta_2 \mathbf{1}\{p = 2\} \right)), \tag{5}$$

so that log-productivity and thus log-wages are given by:

$$\log(\gamma_j(q, p)) = \gamma_j(s) + f(q) \left(\beta_1 + \beta_2 \mathbf{1}\{p = 2\} \right) \tag{6}$$

where $\mathbf{1}\{p = 2\}$ indicates a STEM program graduate. The parameters β_1, and β_2 determine the overall college wage premium and the STEM premium. The function $f(q)$ is increasing and concave in q, implying that productivity gains from college quality eventually saturate. In the quantitative implementation we use an exponential saturation transformation $f(q) =$

$1 - \exp(-\beta_3 q)$, where β_3 governs the speed at which returns to college quality flatten out, and therefore how strongly earnings differ across the observed range of college qualities[3].

Labor markets are segmented by $p \in \{0, 1, 2\}$, and the wage for labor with training p is given by $w(p)$. Realized gross labor income is given by

$$y_j(q,p) = \gamma_j(q,p)w(p)$$

and net labor income of a type (q,p) worker then is $y_j^n(q,p) = y_j(q,p) - T(y_j(q,p))$, and a typical working age budget constraint is given by

$$(1+\tau_c)c_j + a_{j+1} = y_j^n(q,p) + (1 + r(1-\tau_k))a_j \tag{7}$$

Retirement For retired people $j > j_r$, the budget constraint is similar to (7), but labor income $y_j(q,p)$ is replaced by retirement benefits $pen_j(q)$. Households that die prematurely leave accidental bequests that are confiscated by the government and form part of tax revenue.

2.1.2 Preferences

In each period, households derive per-period utility from a period utility function $u(c)$. We further assume that during college $j = 0$ households derive direct utility benefits from attending college (q,p) given by equation (4).

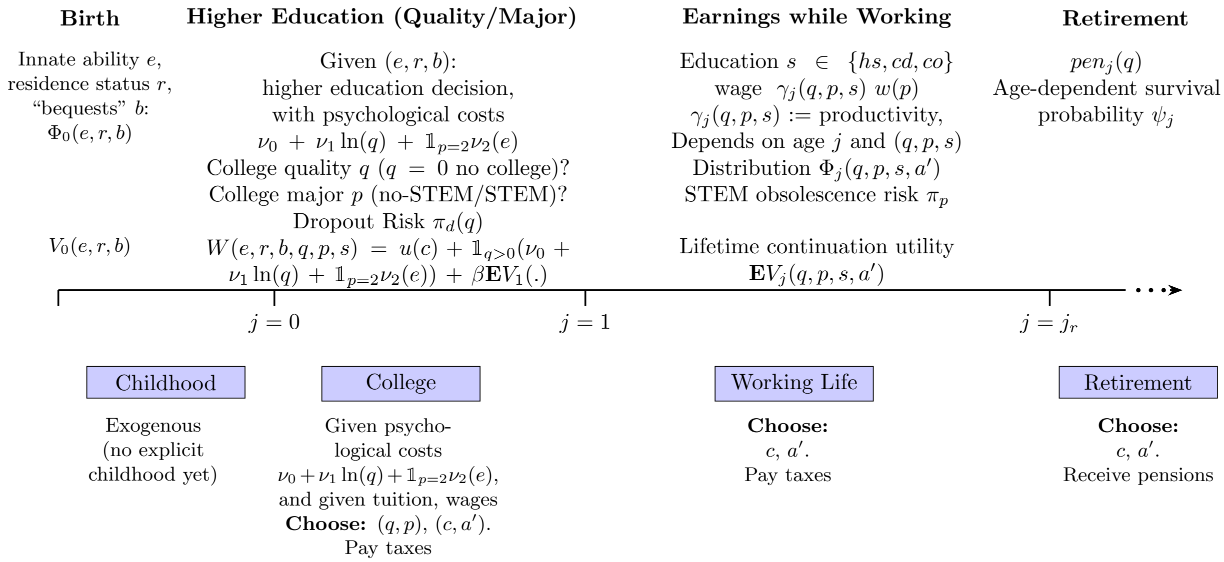
2.2 Timing of Events

At every age $j \geq 1$ households simultaneously make consumption and savings choice. At age $j = 0$, the timing of events is as follows. A household of type (e,r,b) first decides on whether to go to college, and conditional on going, what quality and program combination (q,p) to choose. Finally the household makes consumption-savings choices; at the end of the period the relevant state vector of the household is (e,q,p,a) with associated cross-sectional measure Φ_1.

The timing of events, and consequently the model itself, is summarized succinctly in Figure 2.

[3]In the current version of the paper, labor market productivity does not depend explicitly on innate ability e, and thus e is not currently a state variable for ages $j \geq 1$. At the end of every working age $j \geq 1$ there is a chance that an individual with state $p = 2$ turns into an individual with state $p = 1$. Consequently labor productivity is deterministic, conditional on the type of a worker (e,q,p), but is rendered stochastic by the random transitions of q at the end of the college period to model college dropout risk and p (in every period of working life).

Figure 2: Time Line of the Model



Notes: Time line of the model for period $j = 0$ and for an arbitrary working period $j \in \{1, ..., j_r - 1\}$ and retirement period $j \in \{j_r, ..., J\}$.

2.3 Recursive Formulation of the Household Problem

For all ages $j \geq 1$, households solve a standard consumption-savings problem with labor income being determined by the productivity process $\gamma_j(.)$ and the risk of skill obsolescence if $p = 2$. To capture the risk that STEM-specific skills become obsolete over the working life, we allow the program state p to evolve stochastically: in each period, a worker with $p_j = 2$ transitions to $p_{j+1} = 1$ with probability $\pi_p \in (0, 1)$. Denote the value function at the beginning of model age $j = 1$ as $V_1(e, q, p, a)$. It depends on the education states (q, p), initial ability e (all of which determine labor productivity) as well as asset holdings a (which can be negative if the household borrowed to pay the tuition in college). We spell out the recursive problem(s) giving rise to $V_1(e, q, p, a)$ in Appendix A; this includes determining all age-specific value functions $\{V_j(e, q, p, a)\}$ and associated policy functions $\{(c_j(e, q, p, a)), a_{j+1}(e, q, p, a)\}$, for $j \geq 1$.

We now describe the optimization problems the household solves sequentially during age $j = 0$, backward from the consumption-saving choice to the college attendance, quality and major choice; we spell them out in the main paper as these are the more novel aspects of our model.

2.3.1 The Consumption-Saving Choice

After having made higher education choices (q,p) the relevant state vector of the household is (e,q,p,a) and the household solves the problem:

$$W(e,r,b,q;p) = \max_{c_0,a_1} \{u(c_0) + \beta V_1(e,q,p,a_1)\} \qquad s.t. \tag{8}$$

$$(1+\tau_c)c_0 + a_1 = b + y_0^n - \mathbb{1}_{q>0}(1-\varsigma(b))t(e,r,b;q,p) \tag{9}$$

$$a_1 \geq -\underline{a}_1(q), c_0 \geq 0, q \in \mathcal{Q} \tag{10}$$

where $y_0^n = y_0(1-\mathbb{1}_{q>0}\omega) - T(y_0(1-\mathbb{1}_{q>0}\omega))$ is net labor income during the college phase (which depends on whether the household chooses to attend college or not). The resulting policy functions are denoted by $(c_0(e,r,b,q,p), a_1(e,r,b,q,p))$. It is understood that for $q=0$ (the household not attending college) the p-dimension of the value function is set to $p=0$, standing in for non-college labor.

2.3.2 The College Decision

Working backwards, at the beginning of economic life a household with initial state (e,r,b) chooses what college quality and field of study (q,p) to study. As discussed before, in the theory we think of each (q,p)-combination as a separate college type.

$$V_0(e,r,b) = \max_{q\in\mathcal{Q},p\in\mathcal{P}} \{\mathbb{1}_{q>0}(\nu_0 + \nu_1\ln(q) + \mathbb{1}_{p=2}\nu_2(e)) + W(e,r,b,q,p)\} \tag{11}$$

where $W(e,r,b,q,p)$ was defined in the previous section. We denote the associated optimal college choice functions by $q(e,r,b)$ and $p(e,r,b)$.

2.4 Cross-Sectional Distributions Implied by Household Choices

Given the college choice functions and the initial exogenous probability measure Φ_0 across characteristics (e,r,b) we can now define the cross-sectional distributions at each decision stage for households of age $j=0$, as well as at the beginning of age $j=1$.

In order to prepare the definition of a college market equilibrium we first specify the cross-sectional college quality distribution implied by the household desired college attendance decisions as well as the distribution of initial household characteristics. First, the mass of individuals seeking to attend a college of quality $q \in Q$ and specialization p is given by:

$$\chi(Q,p) = \sum_e \sum_r \int_b \mathbb{1}_{\{q(e,r,b)\in Q, p(e,r,b)=p\}} d\Phi_0(e,r,b) \tag{12}$$

Note that the same equation applies to $(Q,p) = (\{0\},0)$ which determines the set of households not attending college. The mass of households of ability e wanting to attend a college of type (Q,p) is given by

$$\Phi_e(Q,p) = \sum_r \int_b \mathbb{1}_{\{q(e,r,b)\in Q, p(e,r,b)=p\}} d\Phi_0(\{e\},r,b) \tag{13}$$

and thus for all Q with $\chi(Q) > 0$ and for all p the conditional distribution of student ability e (i.e., the share of high-ability students) in quality set Q and major p is given by

$$\Phi(e|Q,p) = \frac{\Phi_e(Q,p)}{\chi(Q,p)}. \tag{14}$$

Of course, the set Q can be a singleton, as long as that singleton has positive mass (as will be the case once we discretize the college quality set $\mathcal{Q}$).

At the end of the college age $j = 0$, or equivalently, at the beginning of age $j = 1$ the remaining state variables are given by (e,q,p,a) and the cross sectional probability measure, for each $e \in \mathcal{E}$, and for all sets $(Q,P,A) \in (\mathbf{B}(\mathcal{Q}), \mathbf{B}(\mathcal{P}), \mathbf{B}(\mathcal{B}))$ with $\{0\} \neq Q$ is given by

$$\Phi_1(\{e\},Q,P,A) = \sum_r \int_b \mathbb{1}_{\{q(e,r,b)\in Q, p(e,r,b)\in P, a_1(b,e,r,q(e,r,b),p(e,r,b))\in A\}} d\Phi_0(\{e\},r,b) \tag{15}$$

For $q = 0$ (households not going to college) it is understood that the college program dimension is irrelevant (and set to $p = 0$):

$$\Phi_1(\{e\},\{0\},\{0\},A) = \sum_r \int_b \mathbb{1}_{\{q(e,r,b)=0, p(e,r,b)=0, a_1(b,e,r,q(e,r,b),p(e,r,b))\in A\}} d\Phi_0(\{e\},r,b) \tag{16}$$

Finally in order to appropriately aggregate the consumption and asset choice at the end of period 0, we need the cross-sectional population measure $\bar{\Phi}_0$ across characteristics (e,r,b,q,p) (since this is what the consumption-savings policies depend upon). This distribution is given by

$$\bar{\Phi}_0(\{e\},\{r\},B,Q,P) = \int_{b\in B} \mathbb{1}_{\{q(e,r,b)\in Q, p(e,r,b)\in P\}} d\Phi_0(\{e\},\{r\},b). \tag{17}$$

where again it is understood that for $q = 0$ the college program dimension is irrelevant.

2.5 College Supply

Having characterized the demand of households for college of different qualities, we now build upon Cai and Heathcote (2022)'s equilibrium model of college supply. To do so, we assume that colleges operate in a competitive environment with free entry into any quality segment $q \in \mathcal{Q} = [0, [q_{\min}, q_{\max}]]$ and program $p \in \mathcal{P}$ of the college market. That is, a college is characterized both by its quality q and the program p it offers.

Our model treats college as an investment good: college quality q and program p raise graduates' future labor market productivity, and therefore the college choice is fundamentally a human capital investment decision. This distinguishes our setting from Cai and Heathcote (2022), whose benchmark model treats college as a consumption good, with important implications for equilibrium properties and the solution algorithm; see Section 3.4 for a detailed discussion.

Suppose a college has a student ability distribution given by $\{\eta_e\}$ where η_e is the share of students of ability e; recall that $e \in \mathcal{E}$ falls in a finite set. Denote by

$$\bar{e} = \sum_e \eta_e e \tag{18}$$

the average student ability in college q. Given $\bar{e}$ colleges decide on instructional spending per student i and produce quality q according to CRS production function

$$q = A_q \left(\frac{\bar{e}}{\bar{e}^{\text{avg}}}\right)^{\theta} \left(\frac{i}{i^{\text{avg}}}\right)^{1-\theta}, \tag{19}$$

where $A_q > 0$ is a TFP parameter, and $\bar{e}^{\text{avg}}$ and i^{avg} denote the mean levels of student ability and instructional spending in the economy, respectively.

In addition to tuition revenue, colleges also receive per student public subsidies $s(e, r; q, p)$ that potentially depend on the quality and program a college delivers and on the characteristics of students the college admits. Conditional on ability e, profit-maximizing colleges strictly prefer to admit only students who generate the most revenue. Let $v(q, p, e) = \max_{b,r} t(e, r, b; q, p) + s(e, r; q, p)$ denote revenue from admitting the highest-revenue students (by resources b and residence status r) of ability level e. Producing programs p also comes at a program specific fixed cost $\kappa(p)$. For a given q, colleges maximize profits:

$$\pi(q, p) = \max_{i, \{\eta_e\}} \left\{ \sum_{e \in E} \eta_e v(q, p, e) - i - \kappa(p) \right\} \tag{20}$$

subject to (19).

When ability can take only two values, with e^l and e^h denoting low and high ability, respectively, we can rewrite the above problem as follows:

$$\pi(q,p) = \max_{i,\eta(e^h)} \left\{ \eta(e^h)[v(q,p,e^h) - v(q,p,e^l)] + v(q,p,e^l) - i - \kappa(p) \right\} \tag{21}$$

subject to

$$q = A_q \left(\frac{\bar{e}}{\bar{e}^{\text{avg}}}\right)^{\theta} \left(\frac{i}{i^{\text{avg}}}\right)^{1-\theta} \tag{22}$$

$$\bar{e} = \eta(e^h)(e^h - e^l) + e^l. \tag{23}$$

Thus, as in the general case, colleges choose per student expenditures and average student ability with the latter being equivalent to choosing the share of high-ability students $\eta(e^h)$ in the case with two ability levels. The first-order conditions to the problem result in the following optimality condition:

$$d(q,p) := -\frac{v(q,p,e^h) - v(q,p,e^l)}{e^h - e^l} = \frac{\theta}{1-\theta}\frac{i}{\bar{e}} > 0 \tag{24}$$

where $d(q,p)$ is the per-unit of ability discount that high-ability students receive relative to low-ability students, on account of the fact that they raise average ability and thus quality of the college. The ability tuition discount for each college type (q,p) is the key equilibrium "pricing" function that the market clearing condition for each college sub-market has to pin down. All other equilibrium elements as well as the qualitative features of the equilibrium then follow from the properties of $d(q,p)$. To see this, first note that the optimality condition (24) can be combined with (23) to determine optimal college inputs as

$$i(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta} \tag{25}$$

$$\bar{e}(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta-1} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta} \tag{26}$$

In terms of intuition, we can think of a given college sub-market (q,p) as being characterized by a low equilibrium quality discount $d(q,p)$ or equivalently, from equation (26) by a high average quality of students $\bar{e}(q,p)$, or (see equation (23)) by a high share of high-ability students,

$$\eta(e^h;q,p) = \frac{\bar{e}(q,p) - e^l}{e^h - e^l} = \frac{\left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta-1} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta} - e^l}{e^h - e^l}. \tag{27}$$

To determine the implications for the tuition schedule, denote by $b(q,p) = v(q,p,e^l)$ the per-student equilibrium revenue a college obtains from a low-ability student. Revenues and tuition for all types are then given by

$$\begin{aligned} v(q,p,e^l) &= b(q,p) && (28)\\ v(q,p,e^h) &= b(q,p) - d(q,p)(e^h - e^l) && (29)\\ t(q,p,e) &= v(q,p,e) - s(q,p,e) \qquad \text{for all } e. && (30) \end{aligned}$$

Since in equilibrium each college in operation makes zero profits, it follows that

$$\begin{aligned} 0 &= \eta(e^h)[v(q,p,e^h) - v(q,p,e^l)] + v(q,p,e^l) - i - \kappa(p) && (31)\\ 0 &= \frac{\bar{e} - e^l}{(e^h - e^l)}[-d(q,p)(e^h - e^l)] + b(q,p) - i(q,p) - \kappa(p) && (32) \end{aligned}$$

which gives as the total revenue of the college from low types (the sticker tuition including government subsidies for low ability students):

$$\begin{aligned} b(q,p) &= \kappa(p) + i(q,p) + \eta(e^h; q,p)d(q,p)(e^h - e^l) && (33)\\ &= \kappa(p) - e^l d(q,p) + \left(\frac{1}{1-\theta}\right)\left[\frac{1-\theta}{\theta}d(q,p)\right]^{\theta} \frac{q}{A_q}\left(\bar{e}^{\text{avg}}\right)^{\theta}\left(i^{\text{avg}}\right)^{1-\theta} && (34) \end{aligned}$$

That is, sticker tuition (including public subsidies) have to cover program specific exogenous per student costs $\kappa(p)$, the optimally chosen discretionary per-student spending $i(q,p)$ as well as the equilibrium discount given to the share $\eta(e^h)$ of high-ability attendees. College revenues and tuition for all types follow from (28) to (30), all as a function of the equilibrium ability discount $d(q,p)$. For future reference, denote the equilibrium measure of college quality and programs by $\chi(.)$, where $\chi(Q,p)$ is the mass of colleges with quality $q \in Q$ that offer program p. The measure of students of ability e^h admitted to program p with quality in Q is given by

$$\int_{q\in Q} \eta(e^h, q, p) d\chi(q,p)$$

From the supply side, the mass of students seeking to attend a college with quality in Q and program p given by

$$\sum_r \int \mathbf{1}_{\{q(e,r,b)\in Q, p(e,r,b)=p\}} d\Phi_0(e,r,b)$$

The discount function $d(q,p)$ adjusts in equilibrium such that for all (Q,p) and for both e these two quantities coincide.

The theoretical properties of the college market equilibrium — namely, independence of equilibrium tuition from students' initial resources, linearity of equilibrium tuition in student ability, and differences in equilibrium tuition schedules and student ability composition across programs ($p = 1$ and $p = 2$) — are derived in Section 3.1 and Appendix B.1.

2.6 Production and the Firm's Problem

Denote by $L(p)$ the aggregate labor efficiency units trained in a given program p (e.g., non-college, college non-stem, college STEM), aggregated across all households across initial wealth b, ability e, residence status r and that attended colleges of quality q. We distinguish between labor trained in different college programs because we posit that the extent to which labor of type $p \in \{0,1,2\}$ can be substituted with capital differs across p. Aggregate college labor of type $p \in \{1,2\}$ in efficiency units is given by

$$L(p) = \sum_e \int_q \int_a \sum_{j=1}^{j_r-1} \mathbb{1}_{\{q(e,r,b)=q, p(e,r,b)=p\}} \gamma_j(q,p) \Phi_j(e, dq, p, da) N_j. \tag{35}$$

Note that this formulation assumes that labor inputs of college graduates from different quality q colleges are perfect substitutes, but that graduates from higher quality colleges are more productive. Likewise, denote aggregate non-college ($p = 0$) labor by

$$L(0) = \sum_e \int_a \sum_{j=1}^{j_r-1} \mathbb{1}_{\{q(e,r,b)=0, p(e,r,b)=0\}} \gamma_j(0,0) \Phi_j(e,0,0,da) N_j. \tag{36}$$

where $\Phi_j(e,0,0,.)$ is the asset probability distribution of individuals of age j and ability e that have chosen not to go to college $(q,p) = (0,0)$.

Denote by $K(p)$ the capital stock used by labor of type p and $\mathbf{K}$ the vector $(K(0), ...K(P))$. Total output in the economy is then produced according to the aggregate constant returns to scale production function

$$Y = F(\mathbf{K}, \mathbf{L}) = \left[\sum_{p \in \{0,\mathcal{P}\}} \lambda(p) y(\Psi K(p), \Gamma(p) L(p))^{\nu} \right]^{\frac{1}{\nu}} \tag{37}$$

$$= \sum_{p \in \{0,\mathcal{P}\}} \left[\lambda(p) \left(\alpha(p) \left(\Psi K(p)\right)^{\rho(p)} + (1-\alpha(p)) \left(\Gamma(p) L(p)\right)^{\rho(p)} \right)^{\frac{\nu}{\rho(p)}} \right]^{\frac{1}{\nu}}. \tag{38}$$

where $y(\Psi K(p), \Gamma(p)L(p))$ is output produced in the "sector" that uses labor from program p, the term $\Gamma(p)$ captures labor-augmenting technological progress (which may differ across p) and the factor Ψ captures capital-augmenting technological change (assumed to be uniform across all capital stocks $K(p)$). The weights $\lambda(p)$ with $\sum_p \lambda(p) = 1$ determine the relative importance (or productivity) of the p-specific sectors, and the elasticity of substitution $\frac{1}{1-\nu}$ between output produced with different labors is determined by the parameter ν. Within each p, production is a CES between capital and labor, with weights $\alpha(p)$ on capital and p-specific substitution elasticity between labor and capital $\sigma(p) = \frac{1}{1-\rho(p)}$. We will calibrate the model such that $\sigma(0) > \sigma(1) > \sigma(2)$, so that the highest ranked college program ($p = 2$, STEM-trained college graduates) is most complementary to capital, and non-college labor is most substitutable with capital. These substitution elasticities will determine the direction and the size of relative and absolute wage effects of changes in the supply of labor induced by changes in demographics and/or education policy or by changes in labor and/or capital productivity designed to mimic potentially skill-biased technological change.

The production technology (38) is operated by a representative firm that operates in perfectly competitive factor and output markets, hires labor efficiency units of the different types p for type-specific wages $w(p)$, rents capital K_t at a rental rate r_t and also decides how to allocate the capital stock across different sectors so that $\sum_p K_t(p) = K_t$. The capital stock depreciates at rate δ. Note that since the firm can freely allocate capital across all uses p, the marginal products across all $K_t(p)$ are equalized and equal to the rental rate plus depreciation, $r_t + \delta$.

Since labor is differentiated by p and outputs across "industries" are not perfect substitutes, there are 3 separate labor markets and associated wages for p-specific labor efficiency units. The firm optimality conditions (as well as the algorithm to compute equilibrium factor prices r_t and $w_t(p)$ from the capital and labor market equilibrium conditions) are provided in equations (121) and (122) in the computational appendix, Section D.3.

2.7 Government

The government provides subsidies $s(e, r; q, p)$ to colleges, proportional to sticker tuition $t(e, r; q, p)$.[4] In addition, the government finances need-based financial aid $\varsigma(b)$, which captures federal and state grants and is specified in detail in Section 4.

The government balances in each period both the general tax-and-transfer system budget and the pension system budget. Expenditures within the general tax-and-transfer system consist of an exogenous stream of non-education-related spending and an endogenous stream

[4] As in Cai and Heathcote (2022), we assume that public subsidies are provided only to in-state students.

of education-related spending, as summarized above. Revenues are generated from taxes on consumption, capital income, and labor income.

We assume that the government has to balance the following per-period budget:

$$\begin{aligned} & \sum_r \sum_e \int_b \mathbb{1}_{\{q(e,r,b)=q, p(e,r,b)=p\}} \left(s(e,r;q,p) + \varsigma(b) t(e,r;q,p) \right) \Phi_0(e,r,db) N_0 \\ = \; & T^c + T^k + T - G \end{aligned} \tag{39}$$

where the left-hand side captures education-related expenditures (public subsidies $s(e,r;q,p)$ and financial aid $\varsigma(b)$), the focus of our analysis. On the net revenue side, T^c denotes consumption tax revenue, T^k denotes capital income tax revenue (including revenue from taxing accidental bequests of deceased households), T denotes labor income tax revenue:

$$\begin{aligned} T^c &= \tau_c \sum_r \sum_e \int_b \int_q \sum_p c_0(e,r,b,q,p) \bar{\Phi}_0(e,r,db,dq,p) N_0 \\ &+ \tau_c \sum_{j=1}^{J} \sum_e \int_q \sum_p \int_a c_j(e,q,p,a) \Phi_j(e,dq,p,da) N_j \\ T^k &= r\tau_k \sum_{j=1}^{J} \sum_e \int_q \sum_p \int_a a_j(e,q,p,a) \Phi_j(e,dq,p,da) N_j \\ &+ \sum_{j_r}^{J} \sum_e \int_q \sum_p \int_a (1-\psi(j)) a_j(e,q,p,a) \Phi_j(e,dq,p,da) N_j \\ T &= \sum_r \sum_e \int_b \int_q \sum_p T(y_0(b,e,r,q,p)) \bar{\Phi}_0(e,r,db,dq,p) N_0 \\ &+ \sum_{j=1}^{J} \sum_e \int_q \sum_p \int_a T(y_j(e,q,p,a)) \Phi_j(e,q,p,a) N_j \end{aligned}$$

and G stands for exogenous non-education related spending G. The pension system budget constraint is also balanced period by period, with benefits paid to current retirees equaling contributions collected from current workers.

2.8 Equilibrium Definition: College Market and Government

A steady state equilibrium is a measure of colleges $\chi(Q,p)$, college tuition $t(b,e,r;q,p)$, average ability of admitted students $\bar{e}(q,p)$, per student college expenditures $i(q,p)$ and profit $\pi(q,p)$ functions of colleges, household choices $c_0(b,e,r)$ and $\{c_j(e,q,a;p)\}_{j=1}^{J}$, $a_1(b,e,r)$ and $\{a_{j+1}(e,q,p,a)\}_{j=1}^{J}$, and $q(e,r,b)$ and $p(e,r,b)$, and government policies that satisfy the following conditions:

1. Given college tuition $t(e, r, b; q, p)$ and government policies (subsidies to colleges, student financial aid and taxes), the household consumption $c_0(e, r, b, q, p)$ and $c_j(e, q, p, a)$, savings $a_1(e, r, b, q, p)$ and $a_{j+1}(e, q, p, a)$ and college quality $q(e, r, b)$ and college major $p(e, r, b)$ choices solve the household optimization problem for all (e, r, b).

2. For all $q > 0$ in the feasible college quality set Ω and all $p \in \mathcal{P}$, the maximal revenue function is determined as $v(q, p, e) = \max_{b,r}\{t(e, r, b; q, p) + s(e, r; q, p)\}$, the college input choices $\bar{e}(q, p)$ and $i(q, p)$ solve the college's profit maximization problem, and $\pi(q, p)$ is the associated profit per student.

3. Zero profits: For all college quality subsets $Q \subset \Omega \setminus \{0\}$ and all $p \in \mathcal{P}$ we have $\int_Q \pi(q, p) d\chi(q, p) = 0$, and $\pi(q, p) \leq 0$ for all $q \in Q$ and $p \in \mathcal{P}$.

4. Goods market clearing:

$$C + \delta K + G + I^{college} + FC^{college} = Y \tag{40}$$

where $I^{college}$ denotes aggregate expenditure of colleges on students, $FC^{college}$ is the total fixed cost paid by colleges, and E denotes the aggregate financial aid provided to students by the government.[5]

5. College market clearing: for all $e \in \mathcal{E}$, all $Q \subseteq \Omega \setminus \{0\}$ and all $p \in \mathcal{P}$

$$\sum_r \int \mathbf{1}_{\{q(e,r,b) \in Q, p(e,r,b)=p\}} d\Phi_0(e, r, b) = \int_{q \in Q} \eta(e, q, p) d\chi(q, p) \tag{41}$$

where

$$(q(e, r, b) = q^*, p(e, r, b) = p^*) \;\Rightarrow\; (b, r) \in \arg\max\{t(e, r, b; q^*, p^*) + s(e, r; q^*, p^*)\} \tag{42}$$

6. The per-period government budget constraint in equation (39) holds in every period.

[5]Written out explicitly, aggregate consumption is given by

$$C = \sum_r \sum_e \int_b \int_q \sum_p c_0(e, r, b, q, p) \bar{\Phi}_0(e, r, db, dq, p) N_0 + \sum_{j=1}^{J} \sum_e \int_q \sum_p \int_a c_j(e, q, p, a) \Phi_j(e, dq, p, da) N_j$$

3 Theory, Computation and Thought Experiments

3.1 Theoretical Results

The college market equilibrium in our model inherits two fundamental properties first established in Cai and Heathcote (2022): (i) equilibrium tuition is independent of students' initial household resources b, and (ii) equilibrium tuition is linear in student ability e. As shown in Appendix B.1, both properties carry over to our setting in which college is an investment good and colleges offer distinct programs $p \in \mathcal{P}$. Building on these properties, the tuition expressions in Section 2.5 (equations (28)–(30)) immediately imply the following results, which are specific to our setting and concern differences in equilibrium tuition schedules and student ability composition across programs ($p = 1$ and $p = 2$).

Proposition 1. *Conditional on the equilibrium quality discount function $d(q,p)$, and for a given college quality q the difference in tuition for STEM programs ($p = 2$) and non-STEM programs ($p = 1$) is given by*

$$\begin{aligned} t(q,2,e) - t(q,1,e) &= \kappa(2) - \kappa(1) - [s(q,2,e) - s(q,1,e)] - [d(q,2) - d(q,1)]\, e^{l} \\ &+ \left(\frac{1}{1-\theta}\right)\left[\frac{1-\theta}{\theta}\right]^{\theta} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta} \left[d(q,2)^{\theta} - d(q,1)^{\theta}\right] \end{aligned}$$

1. *If $d(q,p) = d(q)$, that is, the ability discount is the same across different programs, then for all $q > 0$ and all e*

$$t(q,2,e) - t(q,1,e) = \kappa(2) - \kappa(1) - [s(q,2,e) - s(q,1,e)] \tag{43}$$

   and for schools of a fixed quality q, tuition cost differences by program fully reflect cost differences and differential subsidies, and optimal quality composition $\bar{e}$ and expenditures i are independent of p, for all q.

2. *Now suppose that for fixed quality q, the quality discount is strictly decreasing in p. Then, for a given q STEM programs have a better student quality composition $\bar{e}$ and lower discretionary spending i (but might still have higher cost $\kappa(p) + i$). Tuition differences between programs might be higher or lower than cost differences, and tuition for STEM programs ($p = 2$) might actually be lower than for non-STEM program of the same quality q for some e (possibly even for both e).*

Proposition 2. *For a given college quality q average student quality $\bar{e}$ is strictly increasing in p, that is, strictly higher in STEM colleges than in non-STEM colleges.*

Proof. We want to show that in equilibrium $\bar{e}(q,2) > \bar{e}(q,1)$. From equation (26), this is equivalent to showing that the equilibrium tuition discount satisfies $d(q,2) < d(q,1)$. Suppose not, that is, suppose that $d(q,2) = d(q,1) = d(q)$; if the inequality is reversed, then the argument goes through unchanged. If $d(.)$ is independent of p for fixed q, then from (25) and (26), both programs set identical optimal inputs: $i(q,2) = i(q,1)$ and $\bar{e}(q,2) = \bar{e}(q,1)$. By case 1 of Proposition 1, tuition differences reduce to $t(q,2,e) - t(q,1,e) = \kappa(2) - \kappa(1) - [s(q,2,e) - s(q,1,e)]$, which is independent of e. Since $\nu_2(e)$ is strictly increasing in ability e, high-ability students have a strictly stronger preference for STEM relative to non-STEM than low-ability students. The market clearing condition (41) then requires STEM programs to enroll a strictly higher fraction of high-ability students, i.e., $\bar{e}(q,2) > \bar{e}(q,1)$ – a contradiction. We therefore conclude $d(q,2) < d(q,1)$, and the result follows from (26). □

3.2 Sources of Inefficiency of the Equilibrium and the Scope for Higher Education Subsidies

In our framework, several sources of inefficiency could potentially call for government intervention, and higher education subsidies specifically. First, the presence of borrowing constraints prevent households from smoothing consumption and investment optimally across periods. Although we permit households to borrow to attend college, the potential risk of dropping out of college as well as the risk of post-college low productivity might push households into the borrowing constraint later in life, in turn inducing suboptimal (relative to a complete financial markets benchmark) higher education choice (both at the extensive margin – whether to go to college – and along the intensive margin – what quality college to go to). This friction is especially relevant for low-resource households.

Second, households face uninsurable idiosyncratic risks - most notably, the risk of failing to complete college and the risk of skill obsolescence later in life (e.g., in STEM occupations). In the absence of private insurance markets for such shocks, households must bear them individually, potentially leading to precautionary behavior and inefficient underinvestment in education. Reducing the private costs of attending college through public subsidies mitigates this risk (but of course also induces a moral hazard issue: some individuals might attend college even though private returns are low since the government pays for part of the cost).

Third, government fiscal policies interact with these frictions. In our incomplete markets model, attending a (high-quality) college and/or studying a high potential wage major turns an individual into a high tax payer, a fiscal externality that the individual does not take into account when making higher education decisions. More broadly, subsidies to colleges, financial aid for students, and the progressive labor income tax code might positively impact the distribution of lifetime utilities across individuals. At the same time, these interventions may also distort individual incentives, for instance by affecting equilibrium tuition in the college market or altering the returns to education through the tax-and-transfer system.

3.3 College Market: Mechanisms and Intuition

This subsection summarizes the mechanisms governing tuition schedules and student composition in the college market. To isolate the role of ability composition, which in equilibrium is jointly determined by student demand and college supply, we consider comparative statics within a given college cell (q, p), holding college quality fixed. Thus, the results in Section 5 should be interpreted as partial-equilibrium responses of tuition schedules to changes in the share of high-ability students, rather than as comparative statics of the full equilibrium in which the distribution of college quality itself adjusts.

College market equilibrium. The college-side optimality conditions imply that the equilibrium ability discount satisfies

$$d(q,p) = \frac{\theta}{1-\theta}\frac{i(q,p)}{\bar{e}(q,p)}, \tag{44}$$

where $\bar{e}(q,p)$ denotes average student ability at colleges of type (q, p). With two ability types e^l and e^h, average ability is

$$\bar{e}(q,p) = e^l + \eta(q,p)\Delta e, \tag{45}$$

where $\Delta e = e^h - e^l > 0$ and $\eta(q,p)$ denotes the share of high-ability students attending colleges of type (q, p).

Let $b(q,p)$ denote total revenue obtained from a low-ability student:

$$b(q,p) = \kappa(p) + i(q,p) + \eta(q,p)d(q,p)(e^h - e^l). \tag{46}$$

Revenues by ability are therefore

$$v(q,p,e^l) = b(q,p), \tag{47}$$

$$v(q,p,e^h) = b(q,p) - d(q,p)(e^h - e^l). \tag{48}$$

Substituting into net tuition $t(q,p,e) = v(q,p,e) - s(q,p,e)$ yields

$$
\begin{aligned}
t(q,p,e^l) = \kappa(p) - s(q,p,e^l) + i(q,p) \\
+ \eta(q,p)\, d(q,p)\, \Delta e,
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
t(q,p,e^h) = \kappa(p) - s(q,p,e^h) + i(q,p) \\
- (1-\eta(q,p))\, d(q,p)\, \Delta e.
\end{aligned}
\tag{50}
$$

Comparative statics with respect to ability composition. An increase in the share of high-ability students raises average ability ($\partial \bar{e}/\partial \eta = \Delta e > 0$), which allows colleges to produce a given quality with less instructional spending ($\partial i/\partial \eta < 0$), which in turn reduces the equilibrium ability discount ($\partial d/\partial \eta < 0$). Thus, a larger share of high-ability students increases average ability but lowers both instructional spending and the equilibrium ability discount.

Implications for tuition schedules. Differentiating tuition with respect to $\eta(q,p)$ gives

$$
\frac{\partial t(q,p,e^l)}{\partial \eta} = \frac{\partial i(q,p)}{\partial \eta} + \Delta e \left(d(q,p) + \eta(q,p) \frac{\partial d(q,p)}{\partial \eta} \right), \tag{51}
$$

$$
\frac{\partial t(q,p,e^h)}{\partial \eta} = \frac{\partial i(q,p)}{\partial \eta} + \Delta e \left(d(q,p) - (1-\eta(q,p)) \frac{\partial d(q,p)}{\partial \eta} \right). \tag{52}
$$

The effect of ability composition on tuition operates through its impact on average ability. When $\eta(q,p)$ increases, average ability rises, instructional spending falls, and the equilibrium ability discount shrinks.

For low-ability students both channels reduce tuition, implying $\partial t(q,p,e^l)/\partial \eta < 0$. For high-ability students the two effects work in opposite directions: lower instructional spending reduces tuition, while the reduction in the ability discount increases it. The net effect is therefore ambiguous.

However, the difference between the tuition responses satisfies

$$
\frac{\partial t(q,p,e^h)}{\partial \eta} - \frac{\partial t(q,p,e^l)}{\partial \eta} = -\Delta e \frac{\partial d(q,p)}{\partial \eta} > 0. \tag{53}
$$

Hence, changes in ability composition shift low-ability tuition downward relative to high-ability tuition.

Summary. Three key comparative statics follow from the model:

1. A higher share of high-ability students raises average ability and lowers both instructional spending and the ability discount.

2. When ability composition improves, tuition for low-ability students declines unambiguously, while the response of high-ability tuition is ambiguous.

3. Holding subsidies fixed, an increase in the share of high-ability students reduces the tuition gap between ability types.

These comparative statics provide the intuition for the equilibrium responses of the college market in the quantitative experiments in Section 5.

3.4 Computational Solution of the College Market Equilibrium: College as Investment, Major Heterogeneity, and Credit Frictions

Our treatment of college as an investment good has important implications for both the equilibrium properties of the college market and the computational solution strategy, and it marks a key departure from Cai and Heathcote (2022), whose benchmark model treats college as a consumption good.

Cai and Heathcote (2022) establish two polar results for the investment-good version of their model. First, with a *frictionless* credit, the competitive equilibrium features *no* sorting by income: only ability determines college enrollment (their Proposition 7). Second, when credit frictions are *severe*, the investment model collapses to the consumption model (their Proposition 8), which delivers perfect sorting by income conditional on ability. In the consumption-good (or severe-friction) case, the equilibrium can be solved by a simple and efficient algorithm that exploits the existence of resource thresholds $b^*(q, e)$: for each college quality q and ability level e, there is a cutoff initial wealth above which households enroll, and the college market clears by establishing a mapping from quality q to these wealth thresholds.

Our model occupies the intermediate case: credit market frictions are present but not severe, and college is an investment good. In this intermediate regime neither polar result applies: income sorting is partial, the monotone threshold structure breaks down, and the equilibrium cannot be solved by a simple pass from quality q to income cutoffs. To our knowledge, no existing paper has numerically solved a college market model in this intermediate credit friction case of the investment-good model. Our computational approach is general and efficient enough to handle rich models of the education market that allow for empirically plausible credit frictions, and developing it constitutes an important methodological contribution of the paper. The solution algorithms are described in detail in Appendix D, which contains two algorithms: one for a discrete college quality grid (accommodating a large number of quality levels) and one for a continuous college quality distribution.

3.5 Thought Experiments

To study how demographic forces and education policy interact with the endogenous college market, we conduct three thought experiments. The first experiment isolates the effect of changes in public funding for higher education. The second examines the consequences of demographic transition. The third combines both forces.

Experiment I: Cut of Public Subsidy to Colleges. In the first experiment we remove public subsidies to colleges. This reform raises the net tuition faced by students and allows us to study how the college market adjusts when higher education becomes more expensive. In equilibrium, tuition schedules, enrollment, and the composition of students across college quality and programs adjust endogenously. These first-order changes in college demand and pricing interact with general equilibrium wages and the interest rate and affect public finances through changes in the tax base and the labor income tax rate required to balance the government budget.

Experiment II: Demographic Cliff. In the second experiment we study the consequences of a demographic transition characterized by a persistent decline in fertility. We model this as a transition from an initial steady state with positive population growth to a final steady state with zero population growth. The resulting demographic shift leads to a smaller and older population and therefore alters equilibrium wages and the interest rate through changes in labor supply and capital accumulation. In addition, demographic aging puts pressure on the public pension system. The Social Security budget constraint must be balanced in every period, and in our baseline specification the adjustment occurs through a decline in the pension replacement rate. These general equilibrium adjustments - through both factor prices and pension benefits - affect incentives to invest in higher education and thereby change both college enrollment and the composition of students across college quality and programs. As a result, tuition schedules and ability discounts adjust endogenously in the college market, and the demographic transition also affects the government budget through its impact on the tax base and public finances.

Experiment III: Demographic Cliff with Subsidy Cut. Finally, we combine the two forces by analyzing a demographic transition in an environment in which public subsidies to higher education are removed. This experiment allows us to study how demographic change and fiscal pressures jointly affect equilibrium wages, tuition schedules, and the allocation of students across college quality and majors.

Details on the calibration of the demographic transition and the construction of the population paths used in these experiments are provided in Section 4.1.

4 Calibration

Each period in the model corresponds to four years. Households enter economic life at model age $j_a = 0$ (biological age 18). College education lasts one model period, so all households enter the labor market full time at model age $j_c = 1$ (biological age 22). The statutory retirement age is $j_r = 12$ (biological age 66), and the maximum lifespan is $J = 20$ (biological age ≈101). We calibrate the initial steady state of the model to broadly approximate the U.S. economy in the 2010s.

4.1 Demographic Dynamics

We calibrate the survival probabilities $\{\psi_j\}_{j=0}^{J}$ as the weighted average of female and male survival rates of the population of age 18 and older in year 2025 taken from the Human Mortality Database (HMD). To consistently model the demographic distribution along the transition of the economy to the final steady state, we introduce the notion of age-specific fertility rates into the demographic model, which we calibrate from the year 2025 age-specific fertility rate distribution taken from the United Nations World Population Prospects Database.

To link the population growth rate to the empirical distributions of fertility and survival rates, let the age specific fertility rates in our unisex model be $\bar{\zeta} \cdot \zeta_j$, where we normalize $\sum_j \zeta_j = 1$ so that $\bar{\zeta}$ represents the total fertility rate (TFR). Also note that $\zeta_j > 0$ for $j \in \{\underline{j}, \ldots, \overline{j}\}$ and $\zeta_j = 0$, otherwise. The newborn population in $t+1$ is then

$$N_{t+1,0} = \bar{\zeta} \sum_{j=0}^{J} \zeta_j N_{t,j}. \tag{54}$$

For given $\{\zeta_j, \psi_j\}_{j=0}^{J}$, we get from (1) and (54) the following relationship between the TFR and the population growth rate n follows, see Appendix C.1:

$$\bar{\zeta} = \frac{1}{\sum_{j=0}^{J} \zeta_j \frac{1}{(1+n)^{1+j}} \prod_{i=0}^{j-1} \psi_i}. \tag{55}$$

Observe that for $n = 0$ and $\psi_j = 1, \forall j$, $\bar{\zeta} = 1$. For $\psi_j < 1$ for at least one $j \in \{\underline{j}, \ldots, \overline{j}\}$ we however require $\bar{\zeta} > 1$.

We set the initial steady state population growth rate to the average US adult population growth rate of pre-covid years 2000 to 2018 of $n_{iss}\% = 1\%$, which from equation (55) and given our calibration of $\{\zeta_j, \psi_j\}_{j=0}^{J}$ requires a TFR of $\zeta_{iss} = 1.1427$. The final steady state total fertility rate consistent with a population growth rate of $n_{fss}\% = 0\%$ is $\zeta_{fss} = 1.0107$. We impose this lower fertility rate from year 2008 onwards, which induces the transitional dynamics in the low population growth scenario of Figure 3.

Table 1: Calibration Parameters: First Stage

Symbol	Description	Value	Target/Source
	Demographics		
j_a, j_c, j_r, J	Entry, college exit, retirement, max. age	0, 1, 12, 20	Ages 18, 22, 66, 101
$\{\psi_j\}$	Survival probabilities	—	HMD life tables
	Ability and programs		
E, P	No. of ability types; no. of programs	2; 2	[low, high]; [non-STEM, STEM]
e^h, e^l	High and low ability levels	1.0; 0.375	Cai and Heathcote (2022)
$\mu(e^h)$	Population share, high ability	0.5	Cai and Heathcote (2022)
	College quality production		
A_q	TFP, quality production function	1.77	Mean instructional expenditure
θ	Ability elasticity	0.5	Normalization, Cai and Heathcote (2022)
$\bar{\kappa}$	Avg. college fixed cost	≈\$9,000	Altonji and Zimmerman (2018)
$\kappa(2)/\kappa(1)$	STEM/non-STEM fixed cost ratio	1.71	Altonji and Zimmerman (2018)
	Household preferences, endowments and productivity		
σ, φ	Risk aversion; Frisch elasticity	1; 0.6	—
ℓ, ℓ^{stud}	Avg. and student hours worked	1/3; $\ell/4$	—; NCES
ω	Earnings opportunity cost of college (fraction of y_0 foregone)	20 wks	Cai and Heathcote (2022)
$\bar{b}$	Mean initial resources	\$61,200	Avg. IVT transfer, PSID 2013
ϱ	Resource ratio, high/low ability	1.35	Cai and Heathcote (2022)
σ^b	Variance, initial resource dist.	0.55	Cai and Heathcote (2022)
$\mu(r^i)$	Share of in-state students	0.529	Cai and Heathcote (2022)
π_p	STEM obsolescence prob. (per period)	0.25	Deming and Noray (2020)
$\{\gamma_j(s{=}0)\}$	Age-productivity profile, non-college	—	PSID 1967–2011
$\{\gamma_j(s{=}1)\}$	Age-productivity profile, college	—	PSID 1967–2011
$\underline{a}_1$	College borrowing limit (1 model period = 4 yrs)	\$45,000	Krueger and Ludwig (2016) (\$11,397/yr)
	Aggregate production		
ν	Outer CES elasticity	1	Perfect substitutes
α, δ	Capital share; depreciation rate	33.3%; 5%	—
$\rho(p{=}1)$, $\rho(p{=}2)$	Capital-labor subst. elas., by program	−0.11; −0.43	Caunedo et al. (2023)
$\rho(p{=}0)$	Capital-labor subst. elas., non-college	0.33	Caunedo et al. (2023)
	Government		
$\bar{s}$	Avg. per-student college subsidy rate (in-state)	49%	Cai and Heathcote (2022)
ς_0	Financial aid fraction of sticker tuition	0.21	Cai and Heathcote (2022)
ς_1	Financial aid cap (avg. Pell Grant + state aid)	\$6,870	Cai and Heathcote (2022)
τ_c, τ_k	Consumption and capital income tax rates	5%; 36%	Legislation; Trabandt and Uhlig (2011)
ξ	Labor income tax progressivity	0.18	Heathcote et al. (2017)
τ^p	Social Security payroll tax	12.4%	Legislation
G/Y	Non-education govt. spending/GDP	13.8%	Current value

Notes: First-stage (exogenously set) parameters. HMD = Human Mortality Database; IVT = inter vivos transfers.

Table 2: Calibration Parameters: Second Stage

Symbol	Description	Value	Target/Source
	Household preferences and endowments		
β	Discount factor	0.9882	Interest rate
ν_0	Psych. cost of college attendance	-2.64	College enrollment rate
ν_1	Taste for college quality (warm-glow motive)	0.25	Avg. annual net tuition per student (NCES 2000–2019)
$\nu_2(e)$	Ability-specific STEM disutility	$[-1.65, -1.20]$	STEM shares by ability
ς_d	Slope of graduation rate in quality; $1-\pi_d(q) = \varsigma_d \cdot q$	0.293	Average college dropout rate
β_1, β_2, β_3	Parameters of $\gamma(q, p)$, see (5)	0.55; 0.30; 1.5	Avg. college wage premium; avg. STEM wage premium; earnings gap, top/bottom q quartile
	Aggregate production		
Γ	Wage normalization	1.34	$w(q{=}0) = 1$
	Government		
$\underline{b}_1$	Financial aid eligibility threshold	$\approx 0.7\,\bar{y}$	Share of enrolled students receiving aid
τ_ℓ	Labor income tax level parameter	0.226	Marginal labor income tax rate at avg. earnings

Notes: Second-stage (endogenously calibrated) parameters.

Table 3: Targeted Moments: Empirical Values and Data Sources

Moment	Emp. Value	Data Source
Interest rate (annual)	3.5%	—
College enrollment rate	51%	PSID 2011–2017
Avg. annual net tuition per student (2010 USD, all college types)	\$15,000	NCES 2000–2019
Avg. STEM share among college graduates	24%	NSF 2024 report
STEM share by ability group	—	No direct target; plausible ability gradient
Avg. college wage premium	80%	PSID 2011–2017
College dropout wage premium (rel. to HS graduates)	20%	PSID 2011–2017
Average college dropout rate	50%	NLSY97
Earnings difference between top and bottom quartile of q distribution	98%	Leukhina (2023)
Avg. STEM wage premium	57%	PIAAC 2011 & 2012
Share of enrolled students receiving financial aid	32%	Cai and Heathcote (2022)
Marginal labor income tax rate at avg. earnings	29%	—

Notes: Targeted moments, empirical values, and data sources.

4.1.1 Demographics Dynamics under Demographic Cliff

We now describe how the demographic transition used in the thought experiments II and III is constructed.

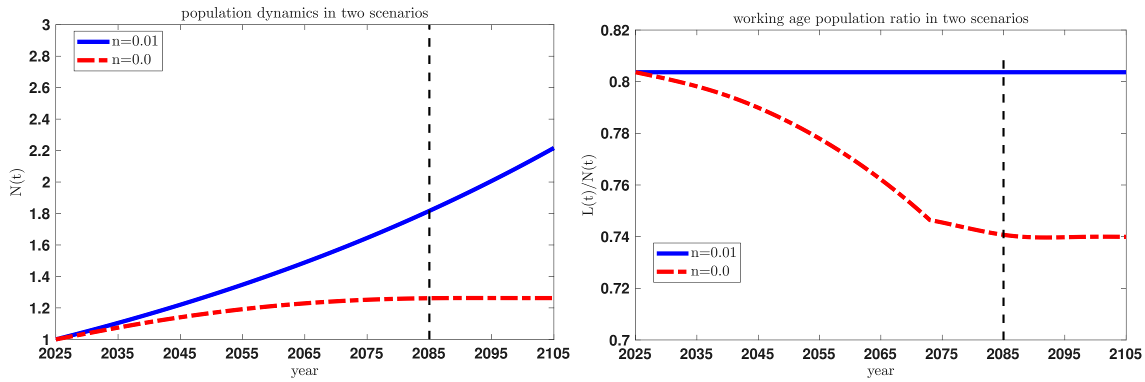
We define the demographic cliff as the drop of the fertility rate in 2008 shown in Figure 1. We focus on the adult population. Those born in year 2007, immediately before the drop of fertility, are of age 18 in year 2025 and thus year 2025 constitutes the base year of our analysis. The year 2025 "benchmark" scenario features a population growth rate of $n_{iss}\% = 1\%$ annually, which we calibrate on the basis of past data, see Section 4.1. We then assume a demographic transition such that in the long run the population converges to a final steady state population distribution with a zero population growth rate, $n_{fss} = 0$, see Figure 1, which shows that the population of age 18 stabilizes in the long-run around 2080 (and so does the total population). We model this transition through a drop in fertility while keeping mortality rates constant at their year 2025 values.

Panel (a) of Figure 3 provides a graphical illustration of the aggregate population transitions, where the blue line represents the dynamics of the adult population in the initial steady state high population growth scenario, in which we normalize the size of the adult population to 1 in year 2025. The red dashed line represents the population in the demographic cliff (low population growth) scenario. In our main experiment we look at a horizon of 60 years and therefore evaluate the drop of the population between the high and the low population growth scenario in year 2085, when the demographic transition is close to a steady state, see Panel (b) of the figure, which shows that the working age to adult population ratio has almost stabilized in 2085. With these assumptions, the size of the population in the low growth population scenario (red dashed line) in 2085 is about 69.44% of the population size achieved in 2085 in the high population growth scenario. Together with the drop of the population growth rate from 1% to 0% these are the three calibration parameters for our demographic cliff experiment of our quantitative steady state comparison, in which we additionally approximate the year 2085 population dynamics in the low population growth scenario by assuming a zero population growth steady state. As the red dashed line of Panel (b) further illustrates, by 2085 the working age to adult population ratio will have dropped from 80.36% to 74.07%, a reduction by 6%p, which shows how the reduction of fertility induces a relative shortage of labor in the economy.

4.2 Households

Preferences. Recall that the psychological cost of attending college depends on child ability and is specified as $\nu_0 + \nu_1 \ln(q) + \mathbb{1}_{p=2}\nu_2(e)$. The parameter ν_0 is calibrated to match the average college enrollment rate, while $\nu_2(e)$ governs the ability-specific shares of students in

Figure 3: Population Dynamics



(a) Population Size (b) Working Age Population Ratio

STEM majors. Finally, ν_1 controls the taste for college quality and is calibrated to match the average net tuition per student (net of grants and scholarships), yielding $\nu_1 = 0.25$. Qualitatively, this parameter captures an additional, non-pecuniary direct utility from college quality beyond the labor market returns of attending a better (and more expensive) college. Households discount utility at rate β, which is calibrated such that the implied steady-state annual interest rate in general equilibrium equals 3.5%.

Endowments, Constraints and Costs. Following Cai and Heathcote (2022), the share of in-state students in the population is set to 0.529.

In our current calibration we follow Cai and Heathcote (2022) and assume two ability types, $e \in \{e^l, e^h\}$. The high-ability type is normalized to $e^h = 1$ and the low-ability type is set to $e^l = 0.375$, as in Cai and Heathcote (2022). The population shares of the two ability types are set to $\mu(e^h) = \mu(e^l) = 0.5$, also following Cai and Heathcote (2022).

The initial resource distribution $F_e(b)$ is log-normal with ability-specific means. The mean initial resources are set to $\bar{b} = \$61{,}200$, corresponding to the average inter-vivos transfer in the PSID (2013). Following Cai and Heathcote (2022), the ratio of mean resources for high- to low-ability households is $\varrho \equiv \bar{b}(e^h)/\bar{b}(e^l) = 1.35$, calibrated to match the ratio of average family income for households in the top versus bottom half of the AFQT score distribution. The standard deviation of log initial resources within each ability group is $\sigma^b = 0.55$, also taken from Cai and Heathcote (2022).

An alternative, richer calibration strategy - which we plan to implement in future work - constructs the ability distribution directly from SAT test score data. Specifically, denoting SAT composite scores by $h \in \mathcal{H} = \{h_1, \ldots, h_H\}$, we build the probability distribution function $\phi(h)$ by assigning the midpoints of SAT composite score bins to the corresponding percentile ranks, see https://blog.prepscholar.com/sat-percentiles-and-score-rankings. The lowest 1 percent of students is assigned an SAT score of $h_1 = 480$ (the midpoint of the 400–660 range), the next 1 percent gets $h_2 = 675$, and so forth, up to $h_H = 1585$ for the top percentile. We then map this fine score distribution into the ability grid $e \in \mathcal{E} = \{e_1, \ldots, e_E\}$, where $E < H$, as follows. First, we normalize the bounds of the support, setting $e_1 = h_1/h_H = 480/1585 \approx 0.3$ and $e_E = 1$. Second, the probability mass at the bounds, $\phi(h_1)$ and $\phi(h_H)$, is assigned to $\phi(e_1)$ and $\phi(e_E)$, respectively. Third, for each interior score $h_j \in \{h_2, \ldots, h_{H-1}\}$, we locate its two neighboring gridpoints e_{i-1} and e_i and distribute the mass $\phi(h_j)$ by linear interpolation: $\Delta\phi(e_{i-1}) = \left(1 - \frac{h_j - e_{i-1}}{e_i - e_{i-1}}\right) \cdot \phi(h_j)$ is added to e_{i-1} and the remainder to e_i.

The graduation rate $1 - \pi_d(q)$ is assumed to be linear in college quality: $1 - \pi_d(q) = \varsigma_d \cdot q$. The slope parameter $\varsigma_d = 0.293$ is calibrated to match an average college dropout rate of 50%, as implied by NLSY97. As an additional validation, the model-implied graduation gradient across college quality tiers is close to the empirical gradient by freshmen SAT scores in IPEDS (2015). After graduation, a worker with STEM degree ($p = 2$) transitions to $p = 1$ with probability $\pi_p = 0.25$ per model period, capturing the risk of STEM skill obsolescence over the working life. While in college, students cannot work full time and, in addition to incurring the psychological cost described above, must also forgo earnings. Consistent with Cai and Heathcote (2022), we assume that students forgo 20 weeks of median weekly earnings of full-time workers aged 16 to 24 while enrolled.

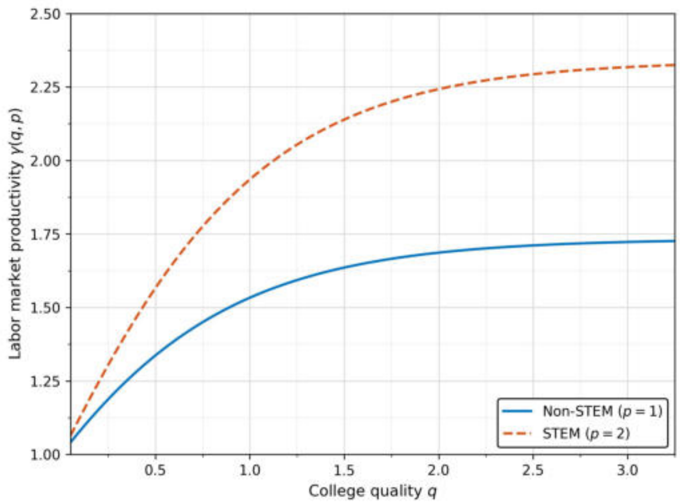
Earnings differences by college quality and major/program of study. Following Hendricks et al. (2021), we classify colleges into four quality types. The lowest quality type consists of two-year community colleges offering transferable associate degrees. The other three types are defined based on the average SAT scores of their freshmen. Type 2 includes four-year institutions with freshmen's average SAT scores in the first tercile of the SAT distribution (least selective public and private colleges). Type 3 includes four-year institutions with scores in the second tercile (many flagship universities and directional schools). Finally, Type 4 includes the most selective four-year institutions with scores in the third tercile (Ivy League, selective private schools, most flagship universities, and many other selective public universities).

As described in Leukhina (2023), after controlling for family income and AFQT scores, graduates from Type 2, Type 3, and Type 4 colleges earn on average 65%, 75%, and 98% more, respectively, than graduates from Type 1 colleges.

Regarding earnings differences by college major, in calibrating $\gamma_j(q, p)$ we target that average earnings of STEM workers are about 57% higher than those of non-STEM workers, based on the Programme for the International Assessment of Adult Competencies (PIAAC, 2011–2012).

The productivity function $\gamma_j(q, p)$ is specified in equation (5). The three parameters β_1, β_2, and β_3 are endogenously calibrated to match the average college wage premium of 80% (PSID, 2011–2017), the average STEM wage premium, and the wage gap between colleges in the bottom and top quartiles of the q distribution, respectively. We obtain $\beta_1 = 0.55$, $\beta_2 = 0.30$, and $\beta_3 = 1.5$, where β_3 enters the saturation function as $f(q) = 1 - \exp(-\beta_3 q)$. Figure 4 illustrates the resulting $\gamma(q, p)$ productivity schedule. Additionally, the age-productivity profiles $\{\gamma_j(s)\}$ are calibrated to PSID earnings data (1967–2011) and are consistent with a wage premium for college dropouts of approximately 20% relative to high-school graduates (PSID, 2011–2017).

Figure 4: Labor Market Productivity



(a) labor productivity by quality q, conditional on major p

Notes: The figure plots labor market productivity $\gamma_j(q, p)$ as a function of college quality q for non-STEM ($p = 1$, solid line) and STEM graduates ($p = 2$, dashed line), as specified in equation (5). Productivity is normalized to 1 for non-college workers ($q = 0$). The saturation function $f(q) = 1 - \exp(-\beta_3 q)$ implies diminishing returns to college quality. Calibrated parameters: $\beta_1 = 0.55$, $\beta_2 = 0.30$, $\beta_3 = 1.5$.

4.3 College Supply

The quality production function (19) involves three parameters: the TFP parameter A_q, the ability elasticity θ, and the program-specific fixed costs $\kappa(p)$. We calibrate them as follows.

The TFP parameter A_q is calibrated to match the empirical mean of instructional expenditure per student, $\bar{\imath}^{\text{avg}}$. Following Cai and Heathcote (2022), the ability elasticity θ is currently set to 0.5 as a normalization. This parameter governs the strength of the club-good mechanism: it directly determines the per-unit ability discount $d(q, p) = \frac{\theta}{1-\theta} \frac{i(q,p)}{\bar{e}(q,p)}$ −

the tuition advantage that accrues to higher-ability students. In future work we plan to calibrate θ to match the ratio of sticker tuition charged by colleges in the top versus the bottom decile of the quality distribution, and to identify ability dispersion $e^h - e^l$ separately from the average generosity of institutional merit-based financial aid observed in the data.

Based on Altonji and Zimmerman (2018), the average fixed cost that colleges bear independently of quality is approximately \$9,000 per student, and the cost of producing graduates in STEM programs is approximately 1.71 times higher than in non-STEM programs, so that $\kappa(2)/\kappa(1) = 1.71$.

4.4 Aggregate Production

We parameterize the aggregate production function specified in equation (38) as follows. In the current quantitative implementation we assume perfect substitutability across labor types at the outer level, i.e. $\nu = 1$; the full derivation for the general CES case is provided in the appendix. The parameters $\rho(p = 0)$, $\rho(p = 1)$ and $\rho(p = 2)$ governing the skill- and major-specific substitution elasticities between capital and labor are based on Caunedo et al. (2023) estimates and are set to 0.33, -0.11 and -0.43, respectively.

Non-college wages are normalized to $w(0) = 1$. We achieve this by scaling Γ using (123) for $p = 0$ to get

$$\Gamma = \frac{\alpha(0)}{1 - \alpha(0)} \cdot \frac{k_t(0)^{\rho(0)-1}}{r_t + \delta}$$

4.5 Government

College subsidies, student financial aid and student loans. Following Cai and Heathcote (2022), the government subsidy $s(e, r; q, p)$ is provided only to in-state students ($r = r^i$) and is set proportional to sticker tuition at rate $\bar{s} = 49\%$, so that $s(e, r^i; q, p) = \bar{s} \cdot t(e, r^i; q, p)$ and $s(e, r^o; q, p) = 0$. We calibrate the student financial aid function $\varsigma(b)$ to capture the two main components of U.S. aid programs: (i) federal need-based grants, primarily Pell Grants, which do not scale with sticker tuition, and (ii) state-level aid programs, which generally do depend on tuition. To this end, we assume that financial aid is provided as a fixed fraction of sticker tuition, but subject to a dollar cap and a resource eligibility threshold:

$$\varsigma(b) = \mathbb{1}\{b \leq \underline{\mathrm{b}}_1\} \cdot \min(\varsigma_1,\ \varsigma_0 \cdot t(e, r; q, p)), \tag{56}$$

where $t(e, r; q, p)$ denotes the sticker tuition faced by a student of ability e, residence status r, college quality q, and program p; ς_0 is the tuition fraction; and ς_1 is the dollar cap. Currently, we assume a single resource threshold $\underline{\mathrm{b}}_1$. All households with family resources below this threshold qualify for financial aid, up to a cap of ς_1 = \$6,870, which corresponds to the average Pell Grant and state-level financial aid reported in Cai and Heathcote (2022). Households with resources above $\underline{b}_1$ do not qualify for aid. The threshold $\underline{b}_1$ is endogenously calibrated such that, in equilibrium, 32% of enrolled students receive financial aid.

Following Krueger and Ludwig (2016), the annual limit on publicly provided student loans is \$11,397, corresponding to a model-period (4-year) borrowing limit of $\underline{a}_1$ = \$45,000.

Taxes and (non-education) transfers. The consumption tax rate is set at 5% (see Mendoza et al. (1994)), and the capital income tax rate is fixed at 36% following Trabandt and Uhlig (2011). The progressive labor income tax code is approximated using a two-parameter tax function as in Heathcote et al. (2017),

$$T(y) = y - \tau_\ell y^{1-\xi},$$

where the progressivity parameter ξ is set to 0.18 - an average estimate across demographic groups in Heathcote et al. (2017) - and the level parameter τ_ℓ is endogenously calibrated so that the implied marginal labor income tax rate at average earnings in the model matches its empirical counterpart.

5 Results

This section studies how demographic forces and public education policy reshape the college market and interact with fiscal policy in general equilibrium. Within the college market, an important equilibrium mechanism operates through the ability composition of the student body. Changes in enrollment alter the share of high-ability students at each college type, which in turn affects instructional spending and the equilibrium ability discount that colleges offer. Because tuition schedules depend directly on this discount, changes in student composition translate into endogenous adjustments in tuition and therefore into further shifts in college demand. Demographic changes affect the college market primarily through their impact on student composition and the resulting adjustment of tuition schedules, in addition to their effects on equilibrium wages and interest rates. Reductions in public college subsidies, in contrast, also operate through a direct price channel, since lower subsidies immediately increase the net tuition that colleges charge students.

In this section we present our basic quantitative results. Section 5.1 displays properties of the steady state equilibrium under the baseline demographic scenario in which the population grows at a constant rate of 1%, the long-run U.S. population growth rate (which factors in immigration) prior to 2008. Section 5.2 then subjects the baseline economy to an unexpected (MIT shock) reduction in public subsidies for higher education; we model a (possibly too) stark reform in which all public subsidies are reduced to zero. In Section 5.3 we turn to the impact of the demographic cliff on the macro economy and the college market, and Section 5.4 discusses the interaction between demographic shifts and public education funding reform. In all scenarios the change in demographics and/or public education subsidies is completely unexpected, and we (currently) compare the long-run (steady state) consequences of these changes.[6]

5.1 Baseline Demographics

In Figure 5 we display the equilibrium allocation in the college market under our benchmark demographic scenario with a population growth rate of 1% (and age-specific mortality rates determined directly from the data). Panel (a) shows the distribution of students across college quality levels. The histogram includes the roughly 50% of high school graduates who do not attend college and therefore select $q = 0$, which appears as the spike at the left end of the distribution.

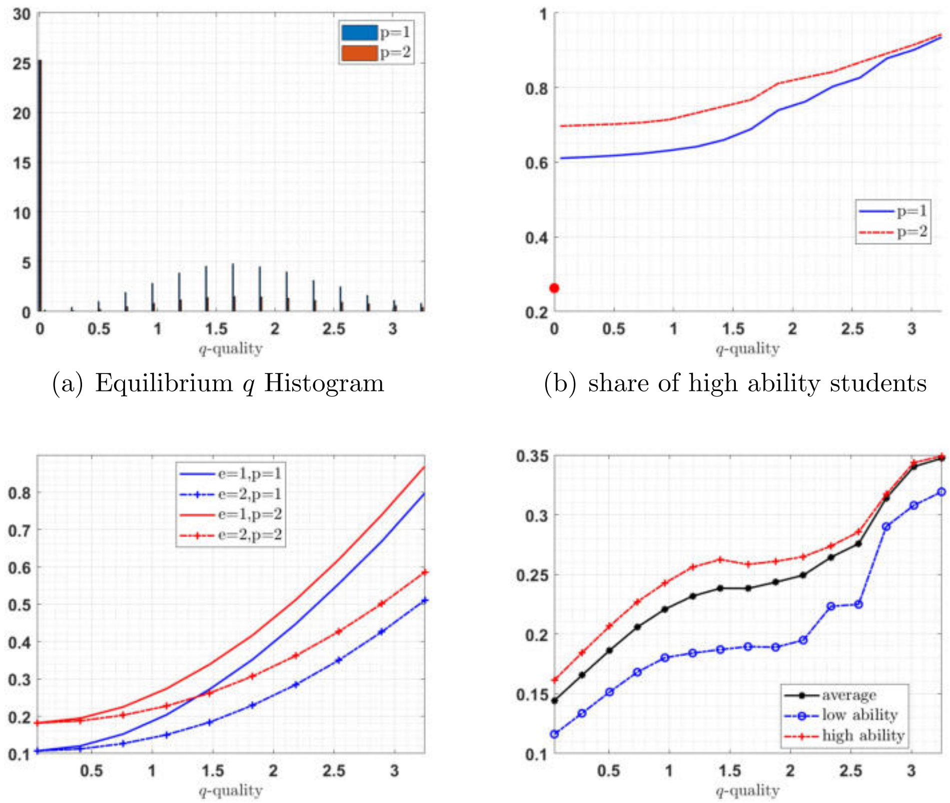
Panel (b) displays the share of high ability ($e = e_h$) students by college quality. It shows that about 80% of high-school students not attending college come from the low-ability group. The remaining share of about 25% consists of high-ability individuals who nevertheless choose not to attend college. This outcome arises because some high-ability individuals start their life with a low b draw (which is a low-probability event but can happen), and are also subject to idiosyncratic taste shocks. Given the tuition schedule, the borrowing limits, the foregone wages while in college and the utility cost of studying, they find it optimal not to attend college despite their high innate ability. The higher the quality of the college, the larger is the ability tuition discount and the share of high-ability students attending colleges of that quality. Given the college quality production function in which per-capita expenditures and average quality enter multiplicatively, the value of admitting a high-ability student is greater at colleges with high per-student expenditures, a property that is reflected in the equilibrium tuition schedule.

Panel (c) of Figure 5 displays the equilibrium tuition schedule. It shows two prominent (but fully expected) features: first, tuition is monotonic in school quality since per-student discretionary spending is one of the two determinants of school quality and colleges have to break even. Second, high ability students receive a tuition discount that compensates them for raising the average quality of the student body, and thus the quality of the college itself, given that average student quality is the second determinant of college quality.[7]

[6]Future versions of this paper will characterize the entire transition paths induced by demographic change and education policy reform. This will be especially crucial for the normative assessment of these changes.

[7]The resulting tuition-ability discount in our model can be interpreted as capturing merit-based institutional financial aid in the U.S.

Figure 5: Baseline: Initial Steady State



(a) Equilibrium q Histogram

(b) share of high ability students

(c) Equil. tuition ($\Delta(\kappa) \approx 0.06$)

(d) share of students in STEM fields, by student ability

Notes: College market equilibrium under the benchmark demographic scenario in which the population grows at a constant rate of 1%.

Panel (d) focuses on the college program (p) or major choice. Recall that studying a STEM field (which in many universities now includes economics) incurs a larger disutility, the more so the less innately able a student ($e = e_l$) is, but carries a higher wage premium in the labor market (and this effect is the stronger the better the attended college, given the log-linear (and thus multiplicative in levels) structure of labor productivity and thus wages in equation (5)). Consequently, the share of STEM students is higher at high-quality colleges, and larger among high ability students (which in turn sort more strongly into high-quality colleges).

Program choice is also influenced by financial constraints. Since the model features borrowing limits, students with limited resources face tighter budget constraints when attending high-quality colleges. For low-ability students in particular, the combination of higher tuition costs and the larger disutility of STEM study makes STEM majors relatively less attractive at the top of the quality distribution. This mechanism generates the slight decline in the

STEM share among low-ability students at the highest-quality colleges. Wealth effects may also play a role because households can save in the model, but the non-monotonicity in the figure is primarily driven by borrowing constraints interacting with the higher disutility of STEM fields.

5.2 Education Subsidy Cuts in General Equilibrium

In this section we quantify how a reduction in public higher education subsidies impacts the college market equilibrium. This exercise is meant to prepare the discussion for the same thought experiment when concurrently the economy is subject to a demographic cliff in Section 5.4. The direct effect of lowering $s(e, r; q)$ uniformly to zero is to increase net tuition for all households seeking to go to college. Of course, in equilibrium the pre-subsidy tuition schedule adjusts, and with it the equilibrium allocation of students across colleges.

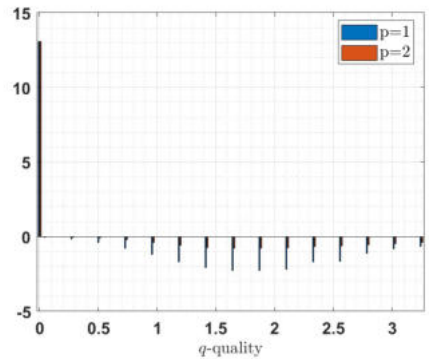
Panel (a) of Figure 6 shows the change in the distribution of college-age individuals across college quality levels q and fields of study p following the subsidy cut. The figure shows a substantial increase – close to 13 percentage points – in the share of households not attending college and a shift away from middle- and high-quality colleges, where tuition levels are highest. Panel (b) displays the resulting change in the ability composition of students. As tuition rises, the college population shifts toward higher-ability students. In the model, colleges offer an ability-based tuition discount, so low-ability students face higher tuition for a given college type. At the same time, low-ability students incur higher utility costs of studying, particularly in STEM fields. In addition, our calibration implies that initial financial resources $b(e)$ are positively correlated with ability, so low-ability students are more likely to be borrowing constrained. Taken together, these forces make college attendance less attractive for low-ability students when net tuition increases, leading to a stronger sorting of high-ability students into college.

Panel (c) plots the percentage change in tuition net of the government subsidy (that is, the tuition students actually pay in equilibrium) as a function of college quality. To interpret this figure, note that if gross tuition charged by colleges had remained unchanged, net tuition would have increased mechanically by

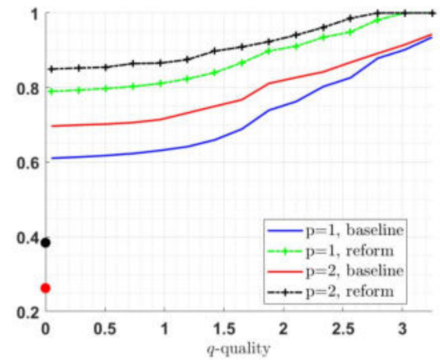
$$\frac{1}{1 - 0.25} - 1 = 33.3\%.$$

As shown in Section 3.3, the improvement in the ability composition of students following the subsidy cut affects tuition through two channels. First, higher average ability reduces the instructional spending required to produce a given quality, lowering tuition for all students. Second, the improvement in student composition reduces the equilibrium ability discount $d(q, p)$, which raises tuition for high-ability students and lowers it for low-ability students.

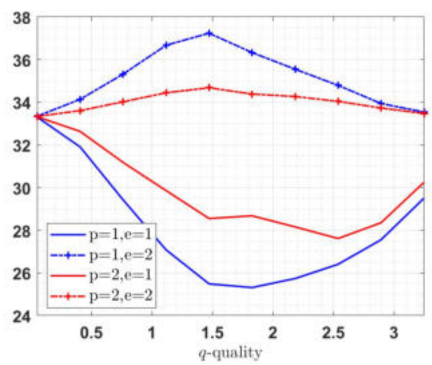
Figure 6: College Funding Cut: Eliminating the 25% Subsidy



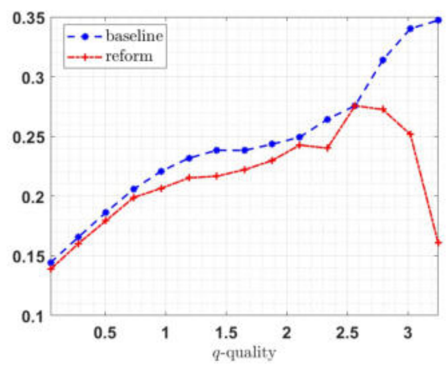
(a) q distribution: change in %p



(b) share of high ability students



(c) %-Δ in equil. tuition schedule



(d) share of students in STEM

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from an average of 25% per student to zero (reflecting a 49% subsidy rate for in-state students, who constitute roughly half the student body). Panel (a): change in the distribution of students across college quality (in percentage points). Panel (b): share of high-ability students. Panel (c): percentage change in the equilibrium net tuition schedule. Panel (d): share of students choosing STEM. All panels plot outcomes as a function of college quality q.

For low-ability students both channels operate in the same direction, so their tuition increases by less than the mechanical 33.3%. For high-ability students the two effects work in opposite directions, but quantitatively the reduction in the ability discount dominates, so their tuition increases slightly more than 33.3%.

Panel (d) shows the share of students choosing STEM majors. Despite the stronger positive selection of students on ability following the subsidy cut, the STEM share declines across the entire college quality distribution and does so particularly strongly at high-quality colleges. The reason is that the increase in net tuition raises the financial cost of attending college. While higher ability would, in isolation, reduce the psychological cost of studying STEM, this effect is dominated by the price channel. As tuition increases, students shift away from the more demanding STEM majors toward less costly alternatives. Because tuition levels are highest at the top of the quality distribution, this mechanism generates the largest decline in the STEM share at high-quality colleges. As a result, both the overall share and the absolute number of STEM students fall substantially following the subsidy cut. To the extent that future technological change increases demand for STEM skills, reductions in higher-education funding may therefore exacerbate STEM shortages in the context of this model.

5.3 The Demographic Cliff and the College Market

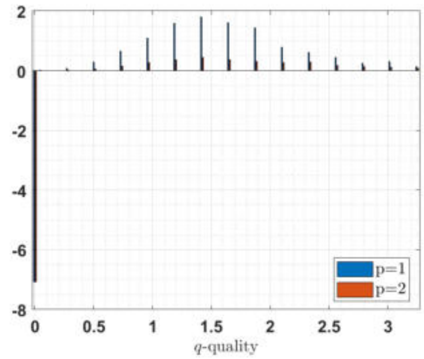
We now consider a shift from a baseline population growth rate of 1% in the initial steady state to 0% in the final steady state. In addition, using our demographic projections (see Section 4.1), we assume that the level of the total population stock shrinks by approximately 30.6%, relative to the benchmark, a drop that corresponds to the value for 2080 in our population model. The latter information is important for the comparison of not only per capita variables, but aggregate levels of economic activity, student body, and fiscal variables across the two demographic scenarios. [8]

Figure 7 illustrates the long-run general equilibrium effects of the demographic shock on the college market and households' higher-education choices. Panel (a) reports percentage-point changes in the equilibrium shares of students choosing a given q (with $q = 0$ denoting non-attendance) and p. The aging shock increases the long-run capital–labor ratio, which lowers the return on physical capital, see Table 4 below. Since capital is more complementary to college-educated than to non-college labor, this change raises the relative return to education and increases equilibrium college enrollment, reducing the non-attendance share by more than six percentage points. In addition, demographic aging puts pressure on the public pension system. In our baseline specification the Social Security budget constraint must be balanced in every period, and the adjustment occurs through a decline in the pension replacement rate. The resulting reduction in expected retirement income further strengthens incentives to invest in higher education. Consistent with these mechanisms, the relative shares of students attending middle-quality colleges also increase.

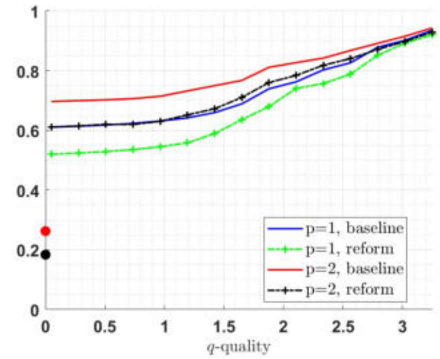
Panel (b) shows that the share of high-ability students declines at low-q colleges. This occurs because the marginal students who switch from non-attendance in the initial steady state to college attendance in the final steady state have, on average, lower ability than the inframarginal students already attending college. At the same time, some high-ability

[8] If we were only interested in per-capita variables, it would not be necessary to compute how the total population changes between the two steady states. However, because we also study aggregate economic activity, the size of the student body, and fiscal variables, the change in the total population stock must be explicitly modeled.

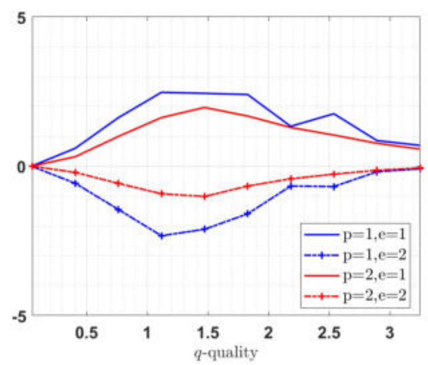
Figure 7: Demographic Cliff



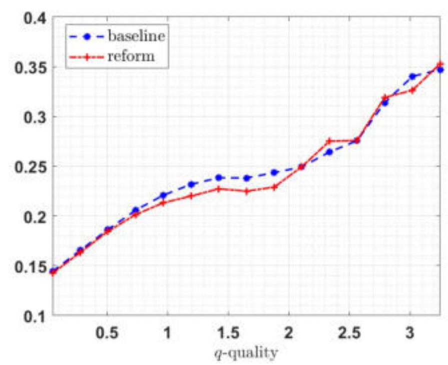
(a) q distribution: change in %p



(b) share of high ability students



(c) %-Δ in equil. tuition schedule



(d) share of students in STEM

Notes: Change in the college market equilibrium in response to a demographic cliff in which the population growth rate falls from 1% to 0%. Panel (a): change in the distribution of students across college quality (in percentage points). Panel (b): share of high-ability students. Panel (c): percentage change in the equilibrium net tuition schedule. Panel (d): share of students choosing STEM. All panels plot outcomes as a function of college quality q.

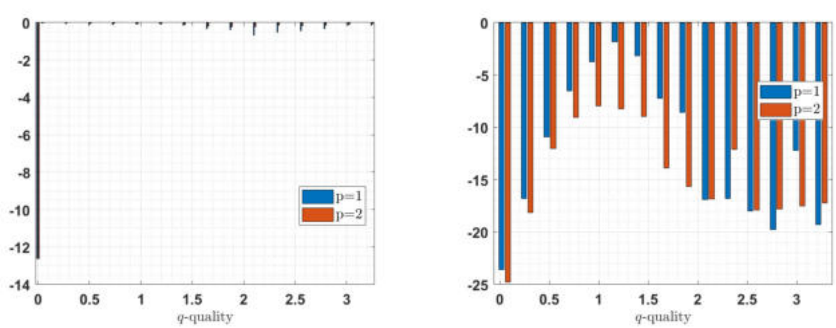
inframarginal students re-sort from low-q to higher-quality colleges, further reducing the high-ability share at the lower end of the college quality distribution. The deterioration in average student ability at middle-quality colleges increases the equilibrium ability discount offered to high-ability students, consistent with the comparative statics derived in Section 3.3, which show that a decline in average ability raises the equilibrium tuition discount for high-ability students.

As shown in panel (d), the STEM share declines at middle-quality colleges following the aging shock. These colleges experience the largest inflow of students, either because previously non-attending marginal students now enroll or because some inframarginal students reallocate from lower-quality colleges. On average, these additional students have lower ability than the students already attending middle-q colleges in the baseline equilibrium. Since

STEM fields are more demanding and involve higher disutility of study, the larger presence of lower-ability students reduces the share of STEM majors at those colleges. This compositional effect also lowers the aggregate STEM share. Because enrollment shifts toward middle-quality colleges, where STEM shares decline, the overall proportion of students choosing STEM falls despite the increase in total college attendance. This result illustrates how shifts in the ability composition of students across college qualities translate into changes in major choice.

Finally, note that the changes in ability composition across the q-distribution lead to noticeable adjustments in equilibrium gross tuition (see panel (c)). The largest changes occur at middle-quality colleges, where enrollment increases the most. These colleges receive an inflow of marginal students who have, on average, lower ability than the inframarginal students attending those colleges in the baseline equilibrium. As a result, average student ability declines at middle-q colleges, which raises the instructional spending required to produce a given level of quality. At the same time, the associated adjustment in the equilibrium ability discount further changes the tuition schedule across ability types. Together, these effects lead to an increase in tuition for low-ability students—up to about 2.5% for non-STEM majors and around 2% for STEM—and a decline in tuition for high-ability students of more than 2% at middle-quality colleges.

Figure 8: Demographic Cliff: Student Enrollment Relative to Baseline Steady State



(a) absolute change in age 18–21 population (baseline = 100)

(b) % change in student population relative to benchmark

Notes: Change in enrollment relative to the baseline steady state at different points in the college quality distribution, in response to a demographic cliff. Panel (a): absolute change in the age 18–21 population as a function of college quality q, with the baseline population normalized to 100 (so values sum to approximately -31, reflecting the 30.6% population decline); for $q = 0$ (non-enrollment), the bar is split into two equal subbars ($p = 1$ and $p = 2$) for visual proportionality only, as there is no major choice for non-college individuals. Panel (b): percentage change in the student population (college attendees only, $q > 0$) relative to the benchmark as a function of college quality q.

The results thus far might suggest that the impact of the demographic cliff on the college market (and, by implication, the labor market and the macroeconomy more broadly) is relatively benign. In percentage terms, the demographic shock raises college attendance and shifts enrollment toward middle-quality colleges as labor scarcity increases the incentives to invest in higher education. However, demographic change also implies a substantial reduction in the absolute number of individuals in the economy—about 30.6% according to our demographic projections. Figure 8 reports the absolute changes in the number of college-age individuals across the college quality distribution relative to the initial high-population benchmark. While the number of individuals not attending college declines sharply, the overall contraction in population leads to much smaller reductions in the absolute number of students. These losses are concentrated primarily at high- and upper-middle-quality colleges, partly reflecting their large baseline enrollment levels. When expressed relative to baseline enrollment, however, the declines are largest at the top and bottom of the quality distribution, whereas middle-quality colleges experience the smallest proportional losses.

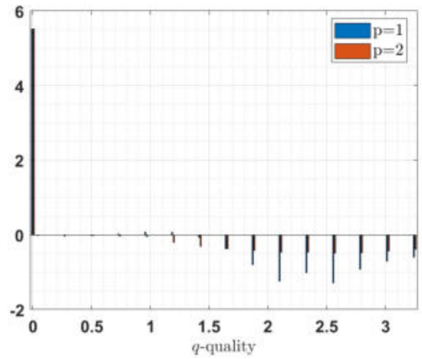
5.4 Public Subsidy Cuts when Population is Aging

Figure 9 shows the college market response when, in addition to the demographic shock just discussed, public funding of colleges is fully withdrawn at the same time, i.e. the subsidy rate is reduced from its baseline level to zero precisely when the demographic cliff hits. Although reality is of course more complex than that, but one could interpret this scenario as the environment the U.S. is currently best described by (or, given the steady state assumption, will be described by in a couple of decades).

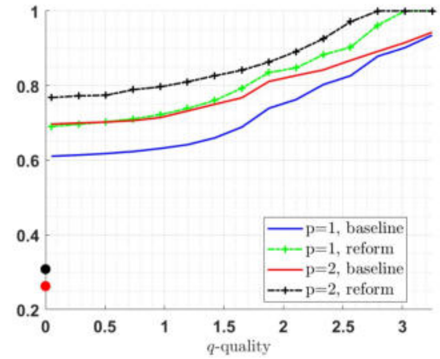
Qualitatively, the responses closely resemble those induced by the pure subsidy cut discussed in Section 5.2. In other words, the favorable effects of the aging shock on the college market in per capita terms (higher enrollment and a shift away from low-quality colleges) are largely undone by the withdrawal of public funding. The magnitudes of the responses are somewhat smaller than in Figure 6, suggesting that the demographic shock making labor scarcer and dearer partially offsets the disincentive effects of the subsidy cut.

Anticipating the discussion of aggregate and fiscal effects in Section 5.5, it is worth noting that the joint experiment combines two adverse forces: a smaller population and a decline in college enrollment relative to the baseline. The result is a particularly severe contraction of macroeconomic aggregates and public finances, one that exceeds the sum of the individual experiments, reflecting the loss of the enrollment-driven fiscal buffer created by the demographic shock.

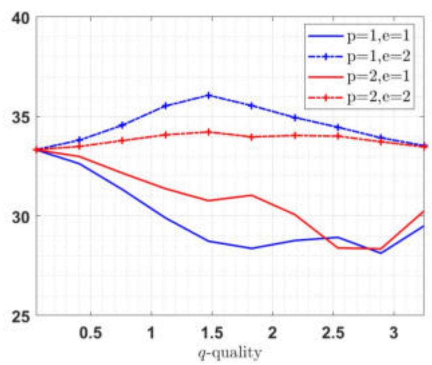
Figure 9: Demographic Cliff & Subsidy Cut



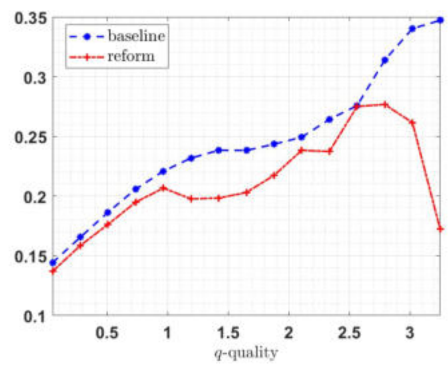
(a) q distribution: change in %p



(b) share of high ability students



(c) %-Δ in equil. tuition schedule



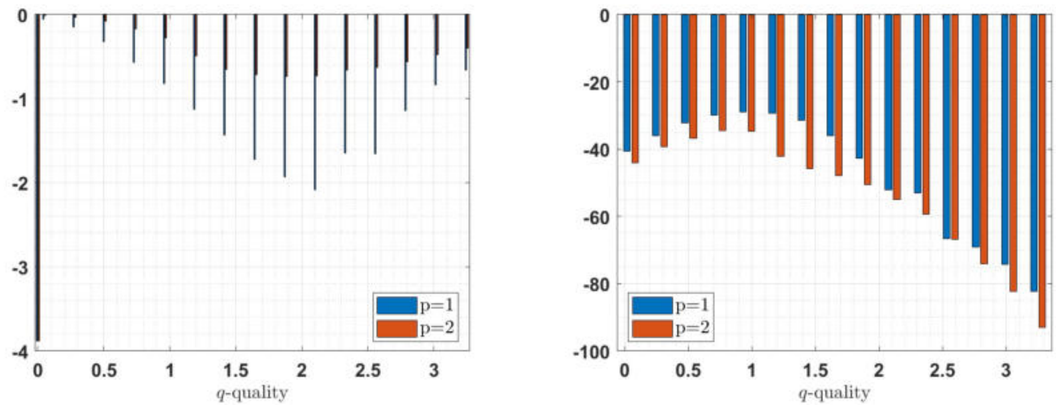
(d) share of students in STEM

Notes: Change in the college market equilibrium in response to a cut in public higher education subsidies from an average of 25% per student to zero (reflecting a 49% subsidy rate for in-state students, who constitute roughly half the student body), in conjunction with the emergence of a demographic cliff. Panel (a): change in the distribution of students across college quality (in percentage points). Panel (b): share of high-ability students. Panel (c): percentage change in the equilibrium net tuition schedule. Panel (d): share of students choosing STEM. All panels plot outcomes as a function of college quality q.

5.5 Aggregate and Fiscal Effects of Demographic Shocks and Subsidy Cuts

The previous subsections examined how the different reforms reshape the college market equilibrium. In this section we summarize the aggregate and fiscal policy implications of the thought experiments discussed thus far. Table 4 contains the impact on interest rates and relative wages, whereas Table 5 and 6 contain the reaction of macroeconomic aggregates and fiscal variables.

Figure 10: Demographic Cliff and Subsidy Cut: Student Enrollment Relative to Baseline Steady State



(a) absolute change in age 18–21 population (baseline = 100)

(b) % change in student population relative to benchmark

Notes: Change in enrollment relative to the baseline steady state at different points in the college quality distribution, in response to a cut in public higher education subsidies in conjunction with a demographic cliff. Panel (a): absolute change in the age 18–21 population as a function of college quality q, with the baseline population normalized to 100 (so values sum to approximately -31, reflecting the 30.6% population decline); for $q = 0$ (non-enrollment), the bar is split into two equal subbars ($p = 1$ and $p = 2$) for visual proportionality only, as there is no major choice for non-college individuals. Panel (b): percentage change in the student population (college attendees only, $q > 0$) relative to the benchmark as a function of college quality q.

Table 4: General Equilibrium: Wages, Interest Rate and Tax Adjustments (% and %p Changes)

Reform	Interest Rate	Non-Coll. Wage	Coll. Non-STEM Wage	Coll. STEM Wage	Lab. Inc. Tax
Exper I	-0.09%p	0.44%	0.48%	0.50%	3.24%p
Exper II	-0.86%p	4.62%	4.92%	5.02%	-0.53%p
Exper III	-0.78%p	4.16%	4.44%	4.54%	3.15%p

Notes: Exper I: elimination of public college subsidies (average subsidy rate cut from 25% per student to zero, reflecting a 49% subsidy rate for in-state students who constitute roughly half the student body). Exper II: demographic cliff (population growth rate falls from 1% to 0%, with total population declining by 30.6%). Exper III: both shocks simultaneously. Interest rate and labor income tax changes are in percentage points (%p); wage changes are in percent (%). All changes are relative to the baseline steady state.

From Table 4 we observe that the demographic shock has a strong impact on equilibrium factor prices. In particular, the aging shock substantially lowers the equilibrium interest rate and raises wages across all skill groups. This reflects the increase in the long-run capital–labor ratio as the population shrinks, which reduces the return to capital and raises the marginal product of labor.

The magnitude of the wage response varies somewhat across skill groups but remains relatively similar across them. In the demographic experiment (Experiment II), wages increase by roughly 4.6% for non-college workers and by about 5% for college-educated workers, with slightly larger increases for STEM graduates. When the demographic shock is combined with the subsidy cut (Experiment III), wage increases remain substantial but are somewhat smaller than in the demographic-only scenario. The reason is that the subsidy cut shifts the financing of college education from the public sector to households. As subsidies are removed, students must finance a larger share of tuition out of pocket, either by borrowing or by reducing saving while studying. This lowers aggregate private saving relative to the demographic-only scenario and leads to a stronger decline in the capital stock. As a result, the increase in the capital–labor ratio is smaller than under the demographic shock alone, so the equilibrium interest rate falls by less and wage gains across skill groups are correspondingly more muted.

The effects on public finances differ across experiments, as summarized in Table 6. In the subsidy-cut experiment (Experiment I), public education spending, as a share of baseline GDP, falls by about 0.91 percentage points. However, the reduction in college attendance also lowers the future tax base, leading to a decline in net revenues, as a share of baseline GDP, of roughly 1.72 percentage points in partial equilibrium. As a result, the labor income tax rate must increase by about 3.24 percentage points in general equilibrium in order to balance the government budget. In contrast, in the demographic experiment (Experiment II), education spending remains unchanged while net revenues, as a share of baseline GDP, fall by about 4.62 percentage points due to the smaller productive working-age population. Nevertheless, higher wages expand the tax base sufficiently so that the labor income tax rate declines slightly, by about 0.53 percentage points.

When both shocks occur simultaneously (Experiment III), the decline in the tax base becomes even more pronounced, with net revenues, as a share of baseline GDP, falling by about 5.98 percentage points. As a result, the labor income tax rate must increase by about 3.15 percentage points — an increase that exceeds the sum of the tax adjustments implied by the subsidy cut and the demographic shock in isolation ($3.24 - 0.53 = 2.71$ percentage points). While the demographic shock alone increases enrollment, the joint experiment drives enrollment substantially below its baseline level. This eliminates the enrollment-driven expansion of the high-skill tax base that previously offset the negative effects of population decline. As a result, the economy is confronted simultaneously with a smaller workforce and a reduced share of high-productivity skilled workers. The demographic shock therefore acts as a fiscal cushion in isolation - one that is undone when subsidies are removed. Once this cushion disappears, both the direct effect of a smaller workforce and the contraction in the skilled tax base materialize jointly, causing the overall fiscal impact to exceed the sum of the two shocks in isolation and requiring a larger tax adjustment.

Table 5: General Equilibrium: Aggregate Variables (% Changes)

Reform	Output	Capital	Aggregate Labor	Output (pc)
Exper I	-10.34%	-12.06%	-10.82%	-10.34%
Exper II	-31.00%	-24.97%	-35.99%	-0.63%
Exper III	-39.90%	-36.78%	-44.91%	-13.45%

Notes: Experiments as defined in the notes to Table 4. Output (pc): output per capita. All entries are percentage changes relative to the baseline steady state.

Table 6: Fiscal Variables (%p Changes)

Reform	Edu. Spend.	Total Rev. (PE)	Lab. Tax Rate Δ (GE)
Exper I	-0.91%p	-1.72%p	3.24%p
Exper II	0.00%p	-4.62%p	-0.53%p
Exper III	-0.91%p	-5.98%p	3.15%p

Notes: Experiments as defined in the notes to Table 4. All entries are in percentage points (%p) relative to benchmark steady-state GDP. Edu. Spend.: on-impact partial equilibrium change in public education expenditure, computed holding enrollment behavior fixed; measures the direct mechanical savings from cutting subsidies (hence identical for Exper I and III, and zero for Exper II). Total Rev. (PE): change in net government revenues — tax revenues *net* of education spending — in partial equilibrium, holding factor prices and the labor income tax rate at their benchmark values; already accounts for the savings from lower education subsidies. Lab. Tax Rate Δ (GE): change in the labor income tax rate in general equilibrium, after all behavioral and price adjustments, required to balance the government budget.

Table 5 shows that the demographic cliff also leads to a large collapse in aggregate labor, capital and output, simply on account of the reduction of the population implied by the new demographic reality, and further reinforced by contractionary fiscal adjustments needed to balance the budget. The decline in public subsidies and associated reduction in college attendance and the shift in the quality distribution to the left magnifies these demographic forces, as the comparison between the second and third row shows.

6 Conclusion

In this paper we have constructed a general equilibrium life cycle model of the college market with heterogeneous colleges, student college quality and major choice, and subsequent labor market outcomes of workers in different occupations. We have applied the model to evaluate the aggregate and distributional consequences of the "demographic cliff" that reduce the number of high-school graduates in the next decades. We have then discussed the consequences of a funding cut towards public higher education akin to the recently imposed measures.

The model simulations show that demographic change fundamentally reshapes the college market and its interaction with public finance. While the "demographic cliff" reduces aggregate output through a smaller population, it simultaneously increases college enrollment rates and partially cushions fiscal pressures by expanding the share of high-skill workers. However, this buffer is fragile: when public subsidies to colleges are removed, enrollment falls substantially below baseline levels and the skill composition deteriorates. As a result, the combined fiscal impact of demographic change and subsidy cuts exceeds the sum of their individual effects.

Our demographic cliff experiments so far were driven by a change in the domestic fertility and therefore domestic population growth rate of the economy. However, significant changes in immigration policy also will have a substantial impact on the age- and the skill distribution of the U.S. population. In a next step we will expand our demographic projections to include this force, with specific attention being placed on the reduction of high-skill migrations and against the backdrop of the quantitative importance of immigrants in STEM fields and innovation.

Second, when conducting our counterfactual experiments thus far, and even though different types of labor have a differential degree of substitutability with capital, we held technology constant (beyond a constant growth trend). Especially when it comes to the importance of STEM education, considering capital-skill-biased technological change (whether exogenously or, eventually, endogenously generated) is important for a full assessment of the changes in the higher education market, and the impact of public funding policies for that market. Whether college should be subsidized, and which programs should be publicly funded, will likely depend on the direction technological progress will take, and we plan to use our environment to shed light on this question. Finally, our analysis thus far compared long-run steady states under different demographic and higher education policy scenarios. Especially a normative assessment of policy reform will require the explicit consideration of transitions, given that demographics and the cross-sectional distribution of higher education attainment are slow-moving variables that will take time to converge to their long run values. We defer this to future versions of this paper.

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Appendix

A Dynamic Programming Problems for $j \geq 1$

To be Completed

B Theoretical Results and Detailed Derivations

B.1 Theoretical Properties of College Market Equilibrium

The two theoretical properties of the college market equilibrium stated below follow directly from Cai and Heathcote (2022), extended to our setting in which college is an investment good – that is, college quality q and program choice p affect graduates' future labor market productivity and earnings – and in which colleges offer distinct programs $p \in \mathcal{P}$ alongside quality q. Since the college supply side operates through the same competitive market structure and CRS quality production function (19), and since neither the investment nature of college nor the program dimension affect the colleges' within-period optimization over inputs $(i, \{\eta_e\})$, the arguments of Cai and Heathcote (2022) go through in our setting with (q, p) replacing q throughout.

Tuition Independence of Initial Resources. Any equilibrium with tuition schedule $\tilde{t}(e, r, b; q, p)$ can be supported by an equivalent schedule that does not depend on initial household resources b, specifically $t^*(e, r; q, p) = v(q, p, e) - s(e, r; q, p)$.

Proof. Fix a college type (q, p) and an ability level e. Let (b^*, r^*) be a revenue-maximizing pair,

$$(b^*, r^*) \in \arg\max_{b,r} \left\{\tilde{t}(e, r, b; q, p) + s(e, r; q, p)\right\},$$

so that $v(q, p, e) = \tilde{t}(e, r^*, b^*; q, p) + s(e, r^*; q, p)$. Since college quality depends only on $\bar{e}$ and i, a college of type (q, p) can always replace a non-revenue-maximizing student of ability e with a revenue-maximizing one without affecting technological feasibility. Therefore, for any $b^{**} \neq b^*$ with $\tilde{t}(e, r^*, b^{**}; q, p) < \tilde{t}(e, r^*, b^*; q, p)$, colleges strictly prefer not to admit type (e, r^*, b^{**}), so college demand for such students at (q, p) is zero. By market clearing, supply of such students at (q, p) must also be zero. We can therefore raise the tuition of type (e, r^*, b^{**}) to $\tilde{t}(e, r^*, b^*; q, p)$ without affecting the equilibrium: supply remains zero since students now face higher tuition, and college demand remains zero since colleges still prefer type (b^*, r^*). Repeating this for all $b \neq b^*$ yields $t(e, r^*, b; q, p) = v(q, p, e) - s(e, r^*; q, p)$ for all b, which is independent of b.

An analogous argument applies to residence status: if $r^{**} \neq r^*$ generates strictly lower total revenue, demand for type (e, r^{**}) at (q, p) is zero, supply is zero by market clearing, and we can raise tuition until $t(e, r^{**}, b; q, p) = v(q, p, e) - s(e, r^{**}; q, p)$ for all b. The resulting schedule $t^*(e, r; q, p) = v(q, p, e) - s(e, r; q, p)$ supports the same equilibrium allocation and is independent of b. □

Revenue Schedule Linear in Ability. Any equilibrium can be supported by a revenue schedule that is linear in student ability e:

$$v(q, p, e) = b(q, p) - d(q, p)\left(e - e^l\right),$$

where $b(q, p) = v(q, p, e^l)$ is the sticker revenue from the lowest-ability student and $d(q, p) > 0$ is the equilibrium ability discount defined in (24).

Proof. By the previous result we work with the income-independent schedule, so it suffices to show $v(q, p, e)$ is linear in e. Fix (q, p) with $\chi(q, p) > 0$ and let $A^+(q, p) = \{e : \eta_e(q, p) > 0\}$ denote the set of ability types with positive enrollment. Suppose $A^+(q, p)$ contains more than one element, with maximum and minimum elements $e_{\max}$ and $e_{\min}$. Define

$$d(q, p) = -\frac{v(q, p, e_{\max}) - v(q, p, e_{\min})}{e_{\max} - e_{\min}}, \qquad b(q, p) = v(q, p, e^l) + d(q, p)(e^l - e_{\min}).$$

We claim $v(q, p, e) = b(q, p) - d(q, p)(e - e^l)$ for all e. Suppose not: then for some $e_j \in A^+(q, p)$ we have $v(q, p, e_j) > b(q, p) - d(q, p)(e_j - e^l)$, i.e., type e_j generates more revenue than the linear schedule would imply. A college could then increase profits by admitting more students of type e_j and substituting out a combination of $e_{\min}$ and $e_{\max}$ students chosen to keep $\bar{e}$ - and hence quality q - unchanged. This deviation is strictly profitable, contradicting zero profits in equilibrium. By a symmetric argument, if $v(q, p, e_j) < b(q, p) - d(q, p)(e_j - e^l)$ for some e_j, then $\eta_{e_j}(q, p) = 0$, so $e_j \notin A^+(q, p)$; we can freely redefine $v(q, p, e_j)$ to lie on the linear schedule without affecting the equilibrium allocation. This establishes linearity for all active ability types and, by the redefinition argument, for all e.

Interior case. When $\eta_e > 0$ for all e (so $A^+(q, p) = \mathcal{E}$), linearity follows directly from the first-order conditions. Letting $\lambda \geq 0$ denote the multiplier on (19) and μ the multiplier on $\sum_e \eta_e = 1$, the first-order condition with respect to η_e gives

$$v(q, p, e) = \mu - \lambda \frac{\theta q}{\bar{e}} e,$$

using $\frac{\partial q}{\partial \bar{e}} = \frac{\theta q}{\bar{e}}$ from (19), which is immediately linear in e. In either case, setting $d(q,p) \equiv \lambda \frac{\theta q}{\bar{e}}$ and using the first-order condition with respect to i, which gives $\lambda = \frac{i}{(1-\theta)q}$, yields $d(q,p) = \frac{\theta}{1-\theta}\frac{i}{\bar{e}}$, consistent with (24). □

C Quantitative Model Calibration Appendix

C.1 Demographic Model

Derivation of Equation (55). Use (1) in (54) to get

$$N_{t+1,0} = \bar{\zeta} \sum_{j=0}^{J} \zeta_j \prod_{i=0}^{j-1} \psi_i N_{t-j,0}. \tag{57}$$

Next, assume a steady state with population growth rate n to get

$$N_{t+1,0} = \bar{\zeta} \sum_{j=0}^{J} \zeta_j \prod_{i=0}^{j-1} \psi_i \frac{N_{t+1,0}}{(1+n)^{1+j}}$$

$$\Leftrightarrow \qquad \bar{\zeta} = \frac{1}{\sum_{j=0}^{J} \zeta_j \prod_{i=0}^{j-1} \psi_i \frac{1}{(1+n)^{1+j}}}.$$

C.2 Asymmetric Effects of Technological Change on College Programs: Mapping College Majors to Tasks

Each college program $p \in \mathcal{P}$ can be characterized by a bundle of tasks — for example, abstract, routine, manual, and interpersonal tasks. We construct a task profile for each program by mapping its curricular content to occupational task requirements, drawing on task measures from sources such as DOT/O*NET. Comparing the task demands of occupations with the curricular content of college majors to establish such a mapping has precedent in the literature; see, for example, Freeman and Hirsch (2008) and Altonji et al. (2014). Relative to an approach based on the occupational destinations of graduates, a curriculum-based mapping more directly captures the skills a program imparts, abstracting from selection into particular occupations.

This mapping allows us to introduce asymmetric effects of technological change across college programs parsimoniously, without altering the model structure. Skill-biased or STEM-biased technological change — including the emergence of AI — manifests as a shift in the productivity of certain task bundles. Because programs differ in their task profiles, such shifts generate differential changes in labor market returns across majors, affecting both college program choices and the equilibrium distribution of college quality. The model's supply and demand structure remains unchanged; only the calibration of program-specific productivity parameters varies across technological scenarios.

D Computational Appendix

D.1 Solution Algorithm: College Market Equilibrium

In the current discrete implementation, college quality q and program p are both discrete, and households choose the pair (q, p) jointly. The key equilibrium pricing object is the per-unit ability discount

$$d(q, p) > 0, \tag{58}$$

which determines the slope of the revenue schedule in ability. By the college-side optimality conditions, revenues are affine in ability:

$$v(q, p, e) = b(q, p) - d(q, p)e, \qquad e \in \mathcal{E}, \tag{59}$$

where $b(q, p)$ is an intercept pinned down by zero profit.

The pass-through from college subsidies to students' net tuition payments is 100% by construction. Accordingly, the household problem is solved using the gross tuition schedule $v(q, p, e)$, while the college-side update is most conveniently expressed in terms of the discount function $d(q, p)$.

Overview. The algorithm has three blocks:

1. solve the household problem after education decisions are made;
2. given a candidate discount schedule $d(q, p)$, recover the college-side objects and construct the gross tuition schedule;
3. solve the household college/program choice problem and update the discount schedule using the implied allocation.

Step 1: Solve the household problem after college entry. Solve the household dynamic programming problem by backward induction from the terminal age J to age $j = j_a$, i.e. up to the initial period at the beginning of which all education-related choices are made.[9] We use an adaptation of the endogenous grid method.

[9] From age $j_a + 1$ onward, the household problem is independent of college-market variables such as tuition and financial aid. The remaining household decisions depend only on labor-market prices and policy parameters.

In period j_a, the household problem is solved on *after-tuition-financial-aid* savings and cash-at-hand/assets grids. After solving for optimal consumption as a function of end-of-period savings:

1. construct beginning-of-period resources net of tuition and financial aid:

$$b^{\text{totnet}} = b - \mathbb{1}_{q>0}\Big[t(b,e,r;q,p)\big(1-\zeta(b)\big)\Big]; \tag{60}$$

2. for each (q,p), ability e, and residence status r, store the age-j_a value function

$$V_{j_a}(b^{\text{totnet}}, e, r; q, p),$$

defined on the endogenous grid for net initial resources b^{totnet}.

Step 2: Iterate on the discount schedule. The iteration computes the fixed point of the mapping

$$d(q,p) \;\rightarrow\; (\bar{e}(q,p), i(q,p), v(q,p,e)) \;\rightarrow\; \pi(q,p|b,e,r) \;\rightarrow\; \bar{e}^{new}(q,p) \;\rightarrow\; d^{new}(q,p).$$

Take as given the age-j_a value functions $V_{j_a}(b^{\text{totnet}}, e, r; q, p)$ computed in Step 1.

1. For each college cell (q,p), initialize a candidate discount

$$d(q,p) > 0.$$

2. Given $d(q,p)$, recover instructional spending and target average student ability from the college-side optimality conditions:

$$i(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta}, \tag{61}$$

$$\bar{e}(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta-1} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta}. \tag{62}$$

Let $\{\eta_e(q,p)\}_{e\in\mathcal{E}}$ denote ability shares consistent with the college-side target ability in cell (q,p). These shares satisfy

$$\sum_{e\in\mathcal{E}} \eta_e(q,p) = 1, \qquad \bar{e}(q,p) = \sum_{e\in\mathcal{E}} \eta_e(q,p)e. \tag{63}$$

These target shares are implied by the current candidate discount schedule $d(q,p)$ and are used to construct the tuition schedule entering the household problem.

3. Using the affine pricing rule

$$v(q,p,e) = b(q,p) - d(q,p)e, \tag{64}$$

and the zero-profit condition

$$\sum_{e\in\mathcal{E}} \eta_e(q,p)v(q,p,e) - i(q,p) - \kappa(p) = 0, \tag{65}$$

recover the intercept

$$b(q,p) = i(q,p) + \kappa(p) + d(q,p)\bar{e}(q,p). \tag{66}$$

Therefore, the full ability-specific revenue schedule is

$$v(q,p,e) = i(q,p) + \kappa(p) + d(q,p)\big(\bar{e}(q,p) - e\big), \qquad e \in \mathcal{E}. \tag{67}$$

Special case: two ability types. When $\mathcal{E} = \{e^l, e^h\}$ with $e^h > e^l$, the college-side allocation is summarized by the share of high-ability students $\eta(e^h; q, p)$. In this case,

$$\bar{e}(q,p) = \eta(e^h; q,p)e^h + \big(1 - \eta(e^h; q,p)\big)e^l, \tag{68}$$

so

$$\eta(e^h; q,p) = \frac{\bar{e}(q,p) - e^l}{e^h - e^l}. \tag{69}$$

The low-type revenue level is

$$b(q,p) = v(q,p,e^l) = \kappa(p) + i(q,p) + \eta(e^h; q,p)d(q,p)(e^h - e^l), \tag{70}$$

and the two ability-specific revenue levels are

$$v(q,p,e^l) = b(q,p), \tag{71}$$
$$v(q,p,e^h) = b(q,p) - d(q,p)(e^h - e^l). \tag{72}$$

Equivalently,

$$v(q,p,e^l) = \kappa(p) + i(q,p) + \eta(e^h; q,p)d(q,p)(e^h - e^l), \tag{73}$$
$$v(q,p,e^h) = \kappa(p) + i(q,p) - \big(1 - \eta(e^h; q,p)\big)d(q,p)(e^h - e^l). \tag{74}$$

4. Let $\Phi(b \mid e)$ denote the exogenous distribution of initial resources before tuition and financial aid. Construct an exogenous grid

$$\mathcal{G}^b = \{b^1, b^2, \ldots, b^n\}$$

over pre-aid resources b.

5. Given the implied tuition schedule $v(q, p, e)$, solve the household's joint college/program choice problem for each state (b, e, r):

   (a) obtain residence-specific net tuition

   $$t(e, r; q, p) = v(q, p, e) - s(e, r; q, p), \tag{75}$$

   where $s(e, r; q, p)$ is the subsidy schedule;

   (b) for each college cell (q, p), compute net initial resources after tuition and aid:

   $$b^{\text{totnet}} = b - \mathbb{1}_{q>0}\Big[t(e, r; q, p)\big(1 - \zeta(b)\big)\Big]; \tag{76}$$

   (c) interpolate the stored age-j_a value function

   $$V_{j_a}(b^{\text{totnet}}, e, r; q, p)$$

   at the corresponding value of b^{totnet};

   (d) solve the household's joint choice over (q, p). In the implementation, the choice is smoothed using Type-I extreme value shocks, so the ex-ante value is

   $$U(b, e, r) = \sigma_\eta \log\left(\sum_{q=0}^{q_{\max}} \sum_{p=1}^{P} \exp\left(\frac{V_{j_a}(b^{\text{totnet}}, e, r; q, p)}{\sigma_\eta}\right)\right); \tag{77}$$

   this step yields joint choice probabilities

   $$\pi(q, p \mid b, e, r).$$

6. Using the exogenous distribution of household states and the implied joint choice probabilities, compute enrollment in each college cell (q, p), denoted by $\chi(q, p)$. For each

active college cell, compute the associated ability shares

$$\eta_e^{new}(q,p) = \frac{\sum_r \int \pi(q,p \mid b,e,r)\, d\Phi_0(b,e,r)}{\chi(q,p)}, \qquad e \in \mathcal{E}, \tag{78}$$

where

$$\chi(q,p) = \sum_{e\in\mathcal{E}} \sum_r \int \pi(q,p \mid b,e,r)\, d\Phi_0(b,e,r). \tag{79}$$

Hence, average student ability in cell (q,p) is

$$\bar{e}^{new}(q,p) = \sum_{e\in\mathcal{E}} \eta_e^{new}(q,p)e. \tag{80}$$

7. In equilibrium, the household allocation must be consistent with the college-side objects $\bar{e}(q,p)$, $i(q,p)$, and $v(q,p,e)$ implied by $d(q,p)$. The discount schedule therefore adjusts until the demand-implied composition matches the college-side target.
   Update the discount schedule using the college-side optimality condition

$$d^{\text{new}}(q,p) = \frac{\theta}{1-\theta} \frac{i^{new}(q,p)}{\bar{e}^{new}(q,p)}, \tag{81}$$

where updated instructional spending is implied by the quality production function (using the demand-side average ability):

$$i^{new}(q,p) = \left(\frac{q}{A_q}\right)^{\frac{1}{1-\theta}} i^{\text{avg}} \left(\bar{e}^{\text{avg}}\right)^{\frac{\theta}{1-\theta}} \left(\bar{e}^{new}(q,p)\right)^{-\frac{\theta}{1-\theta}}. \tag{82}$$

Equivalently, substituting out $i^{new}(q,p)$,

$$d^{\text{new}}(q,p) = \frac{\theta}{1-\theta} \frac{i^{\text{avg}}}{\bar{e}^{\text{new}}(q,p)} \left(\frac{q}{A_q}\left(\frac{\bar{e}^{\text{avg}}}{\bar{e}^{\text{new}}(q,p)}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \tag{83}$$

$$= \frac{\theta}{1-\theta}\left(\frac{q}{A_q}\right)^{\frac{1}{1-\theta}} \left(\bar{e}^{\text{avg}}\right)^{\frac{\theta}{1-\theta}} \imath^{\text{avg}} \left(\bar{e}^{\text{new}}(q,p)\right)^{-\frac{1}{1-\theta}}. \tag{84}$$

Special case: two ability types. When $\mathcal{E} = \{e^l, e^h\}$, the ability shares are summarized by the high-ability share

$$\eta^{new}(e^h;q,p) = \frac{\sum_r \int \pi(q,p \mid b,e^h,r)\, d\Phi_0(b,e^h,r)}{\chi(q,p)}, \tag{85}$$

and

$$\bar{e}^{new}(q,p) = \eta^{new}(e^h;q,p)e^h + \big(1 - \eta^{new}(e^h;q,p)\big)e^l. \tag{86}$$

Therefore,

$$d^{\text{new}}(q,p) = \frac{\theta}{1-\theta}\left(\frac{q}{A_q}\right)^{\frac{1}{1-\theta}} (\bar{e}^{\text{avg}})^{\frac{\theta}{1-\theta}}\, i^{\text{avg}} \left[\eta^{\text{new}}(e^h;q,p)e^h + \big(1-\eta^{\text{new}}(e^h;q,p)\big)e^l\right]^{-\frac{1}{1-\theta}}. \tag{87}$$

8. Compute the sup norm distance in the discount schedule:

$$\Delta = \max_{q,p} \left| d^{\text{new}}(q,p) - d(q,p) \right|. \tag{88}$$

If Δ exceeds a chosen tolerance threshold, update the discount schedule and return to Step 2(b). Otherwise, the algorithm has converged.

Remark. The key feature of the implementation is that, given a candidate discount schedule $d(q,p)$, the college-side optimality conditions determine instructional spending $i(q,p)$, the college-side target for average student ability $\bar{e}(q,p)$, and the affine ability-specific tuition schedule $v(q,p,e)$. The household block then determines the allocation of students across college cells (q,p), and the discount function adjusts until these objects are jointly consistent with market clearing in every college sub-market.

D.2 Solution Algorithm: College Market Equilibrium with Continuous College Quality

In the continuous implementation, college quality q is continuous while program p remains discrete, so colleges are indexed by pairs (q,p). As in the discrete implementation, the key equilibrium pricing object is the per-unit ability discount

$$d(q,p) > 0, \tag{89}$$

which determines the slope of the revenue schedule in ability. By the college-side optimality conditions, revenues are affine in ability:

$$v(q,p,e) = b(q,p) - d(q,p)e, \qquad e \in \mathcal{E}, \tag{90}$$

where $b(q,p)$ is an intercept pinned down by zero profit.

To obtain a finite-dimensional fixed-point problem, we approximate the discount schedule $d(q,p)$ using cubic B-splines in q. For each program p,

$$d(q,p) = \sum_{m=1}^{M} \theta_{m,p} B_m(q), \tag{91}$$

where $\{B_m(q)\}_{m=1}^{M}$ are spline basis functions defined on the admissible quality interval $[\underline{q}, \overline{q}]$.

The pass-through from college subsidies to students' net tuition payments is 100% by construction. Accordingly, the household problem is solved using the gross tuition schedule $v(q,p,e)$, while the college-side update is most conveniently expressed in terms of the discount function $d(q,p)$.

Quality grids. The computation uses two separate grids for quality:

1. a coarse grid $\mathcal{Q}^c$, used to solve and store household value functions;
2. a dense grid $\mathcal{Q}^d$, used to solve the household college choice problem and to update the discount schedule.

Objects stored on the two grids. The dense quality grid $\mathcal{Q}^d$ is used only for college-market objects needed to update the discount schedule, namely the college-cell enrollment measure $\chi(q,p)$, the demand-implied ability shares $\eta_e(q,p)$, and the corresponding average ability $\bar{e}(q,p)$. All other household-side objects — including value functions, policy functions, and the household cross-sectional distribution over individual states — are stored only on the coarse quality grid $\mathcal{Q}^c$. Hence, the dense grid is used only within the college-choice block, whereas the rest of the household problem is represented on the coarse grid.

Overview. The algorithm has three blocks:

1. solve the household problem after education decisions are made;
2. given a candidate discount schedule $d(q,p)$, recover the college-side objects and construct the gross tuition schedule;
3. solve the household college/program choice problem on the dense grid and update the discount schedule using the implied allocation.

Step 1: Solve the household problem after college entry. Solve the household dynamic programming problem by backward induction from the terminal age J to age $j = j_a$, i.e. up to the initial period at the beginning of which all education-related choices are made.[10] We use an adaptation of the endogenous grid method.

In period j_a, the household problem is solved on *after-tuition-financial-aid* savings and cash-at-hand/assets grids. After solving for optimal consumption as a function of end-of-period savings:

1. construct beginning-of-period resources net of tuition and financial aid:

$$b^{\text{totnet}} = b - \mathbb{1}_{q>0}\Big[t(b,e,r;q,p)\big(1-\zeta(b)\big)\Big]; \tag{92}$$

2. for each $q \in \mathcal{Q}^c$, program p, ability e, and residence status r, store the age-j_a value function

$$V_{j_a}(b^{\text{totnet}}, e, r; q, p),$$

defined on the endogenous grid for net initial resources b^{totnet}.

Step 2: Iterate on the discount schedule. Take as given the age-j_a value functions $V_{j_a}(b^{\text{totnet}}, e, r; q, p)$ computed in Step 1.

1. For each program p, initialize spline coefficients θ_p, which imply a candidate discount schedule

$$d(q,p) = \sum_{m=1}^{M} \theta_{m,p} B_m(q), \qquad q \in \mathcal{Q}^d. \tag{93}$$

2. Given $d(q,p)$, recover instructional spending and target average student ability from the college-side optimality conditions:

$$i(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta}, \tag{94}$$

$$\bar{e}(q,p) = \left[\frac{1-\theta}{\theta} d(q,p)\right]^{\theta-1} \frac{q}{A_q} \left(\bar{e}^{\text{avg}}\right)^{\theta} \left(i^{\text{avg}}\right)^{1-\theta}. \tag{95}$$

[10]From age $j_a + 1$ onward, the household problem is independent of college-market variables such as tuition and financial aid. The remaining household decisions depend only on labor-market prices and policy parameters.

Let $\{\eta_e(q,p)\}_{e\in\mathcal{E}}$ denote ability shares consistent with the college-side target ability in cell (q,p). These shares satisfy

$$\sum_{e\in\mathcal{E}} \eta_e(q,p) = 1, \qquad \bar{e}(q,p) = \sum_{e\in\mathcal{E}} \eta_e(q,p)e. \tag{96}$$

These target shares are implied by the current candidate discount schedule $d(q,p)$ and are used to construct the tuition schedule entering the household problem.

3. Using the affine pricing rule

$$v(q,p,e) = b(q,p) - d(q,p)e, \tag{97}$$

and the zero-profit condition

$$\sum_{e\in\mathcal{E}} \eta_e(q,p)v(q,p,e) - i(q,p) - \kappa(p) = 0, \tag{98}$$

recover the intercept

$$b(q,p) = i(q,p) + \kappa(p) + d(q,p)\bar{e}(q,p). \tag{99}$$

Therefore, the full ability-specific revenue schedule is

$$v(q,p,e) = i(q,p) + \kappa(p) + d(q,p)\big(\bar{e}(q,p) - e\big), \qquad e \in \mathcal{E}. \tag{100}$$

Special case: two ability types. When $\mathcal{E} = \{e^l, e^h\}$ with $e^h > e^l$, the college-side allocation is summarized by the share of high-ability students $\eta(e^h; q,p)$. In this case,

$$\bar{e}(q,p) = \eta(e^h;q,p)e^h + \big(1 - \eta(e^h;q,p)\big)e^l, \tag{101}$$

so

$$\eta(e^h;q,p) = \frac{\bar{e}(q,p) - e^l}{e^h - e^l}. \tag{102}$$

The two ability-specific revenue levels are

$$v(q,p,e^l) = \kappa(p) + i(q,p) + \eta(e^h;q,p)d(q,p)(e^h - e^l), \tag{103}$$

$$v(q,p,e^h) = \kappa(p) + i(q,p) - \big(1 - \eta(e^h;q,p)\big)d(q,p)(e^h - e^l). \tag{104}$$

4. Let $\Phi(b \mid e)$ denote the exogenous distribution of initial resources before tuition and financial aid. Construct an exogenous grid

$$\mathcal{G}^b = \{b^1, b^2, \ldots, b^n\}$$

over pre-aid resources b.

5. Given the implied tuition schedule $v(q,p,e)$, solve the household's college/program choice problem for each state (b,e,r) on the dense quality grid:

(a) obtain residence-specific net tuition

$$t(e,r;q,p) = v(q,p,e) - s(e,r;q,p), \tag{105}$$

where $s(e,r;q,p)$ is the subsidy schedule;

(b) for each dense-grid quality point $q \in \mathcal{Q}^d$ and each program p, compute net initial resources after tuition and aid:

$$b^{\text{totnet}} = b - \mathbb{1}_{q>0}\Big[t(e,r;q,p)\big(1-\zeta(b)\big)\Big]; \tag{106}$$

(c) interpolate the stored age-j_a value function

$$V_{j_a}(b^{\text{totnet}}, e, r; q, p)$$

at the corresponding value of b^{totnet}, interpolating across both net resources and quality q from the coarse grid $\mathcal{Q}^c$ to the dense grid $\mathcal{Q}^d$;

(d) conditional on each quality level q, solve the program choice problem. In the implementation, program choice is smoothed using Type-I extreme value shocks:

$$W(b,e,r;q) = \sigma_\eta \log\left(\sum_{p=1}^{P} \exp\left(\frac{V_{j_a}(b^{\text{totnet}}, e, r; q, p)}{\sigma_\eta}\right)\right). \tag{107}$$

This step yields conditional program choice probabilities

$$\pi(p \mid b, e, r; q).$$

(e) given $\{W(b,e,r;q)\}_{q\in\mathcal{Q}^d}$, solve the college-quality choice problem by direct comparison on the dense grid:

$$q^*(b,e,r) = \arg\max_{q\in\mathcal{Q}^d} W(b,e,r;q). \tag{108}$$

Since quality is treated as continuous and approximated on a dense grid, no additional extreme-value shocks are introduced at the quality-choice stage.

6. Using the exogenous distribution of household states and the implied policy functions, compute enrollment in each college cell (q,p), denoted by $\chi(q,p)$. For each active college cell, compute the associated demand-implied ability shares

$$\eta_e^{\text{new}}(q,p) = \frac{\sum_r \int \mathbb{1}\{q^*(b,e,r)=q\}\pi(p \mid b,e,r;q)\, d\Phi_0(b,e,r)}{\chi(q,p)}, \qquad e\in\mathcal{E}, \tag{109}$$

where

$$\chi(q,p) = \sum_{e\in\mathcal{E}}\sum_r \int \mathbb{1}\{q^*(b,e,r)=q\}\pi(p \mid b,e,r;q)\, d\Phi_0(b,e,r). \tag{110}$$

Hence, demand-implied average student ability in cell (q,p) is

$$\bar{e}^{\text{new}}(q,p) = \sum_{e\in\mathcal{E}} \eta_e^{\text{new}}(q,p)e. \tag{111}$$

Only the college-market aggregates $\chi(q,p)$, $\eta_e^{\text{new}}(q,p)$, and $\bar{e}^{\text{new}}(q,p)$ are retained on the dense grid after solving the household problem.

7. In equilibrium, the demand-implied composition must coincide with the college-side target composition in every college cell. The discount schedule therefore adjusts until the demand-implied average ability matches the college-side target. Using the demand-implied average ability, update instructional spending via the quality production function:

$$i^{new}(q,p) = \left(\frac{q}{A_q}\right)^{\frac{1}{1-\theta}} i^{\text{avg}} \left(\bar{e}^{\text{avg}}\right)^{\frac{\theta}{1-\theta}} \left(\bar{e}^{new}(q,p)\right)^{-\frac{\theta}{1-\theta}}. \tag{112}$$

Then update the discount schedule using the college-side optimality condition:

$$d^{\text{new}}(q,p) = \frac{\theta}{1-\theta} \frac{\bar{\imath}^{\text{avg}}}{\bar{e}^{\text{new}}(q,p)} \left(\frac{q}{A_q} \left(\frac{\bar{e}^{\text{avg}}}{\bar{e}^{\text{new}}(q,p)} \right)^{\theta} \right)^{\frac{1}{1-\theta}} \tag{113}$$

$$= \frac{\theta}{1-\theta} \left(\frac{q}{A_q} \right)^{\frac{1}{1-\theta}} \left(\bar{e}^{\text{avg}} \right)^{\frac{\theta}{1-\theta}} \, 1^{\text{avg}} \left(\bar{e}^{\text{new}}(q,p) \right)^{-\frac{1}{1-\theta}} . \tag{114}$$

Two-type update. When $\mathcal{E} = \{e^l, e^h\}$, the ability shares are summarized by the high-ability share

$$\eta^{\text{new}}(e^h; q, p) = \frac{\sum_r \int \mathbb{1}\{q^*(b, e^h, r) = q\} \pi(p \mid b, e^h, r; q) \, d\Phi_0(b, e^h, r)}{\chi(q,p)}, \tag{115}$$

and

$$\bar{e}^{\text{new}}(q,p) = \eta^{\text{new}}(e^h; q, p) e^h + \big(1 - \eta^{\text{new}}(e^h; q,p)\big) e^l. \tag{116}$$

Therefore,

$$d^{\text{new}}(q,p) = \frac{\theta}{1-\theta} \left(\frac{q}{A_q} \right)^{\frac{1}{1-\theta}} \left(\bar{e}^{\text{avg}} \right)^{\frac{\theta}{1-\theta}} \, \bar{\imath}^{\text{avg}} \left[\eta^{\text{new}}(e^h; q,p) e^h + \big(1 - \eta^{\text{new}}(e^h; q,p)\big) e^l \right]^{-\frac{1}{1-\theta}} . \tag{117}$$

8. For each program p, update the spline coefficients by projecting the dense-grid discount update onto the B-spline basis. Let $\mathcal{Q}^{d,\text{act}}(p) \subseteq \mathcal{Q}^d$ denote the set of dense-grid quality points with non-negligible enrollment mass in program p. Then

$$\theta_p^{\text{new}} = \arg\min_{\theta_p} \sum_{q \in \mathcal{Q}^{d,\text{act}}(p)} w(q,p) \left[d^{\text{new}}(q,p) - \sum_{m=1}^{M} \theta_{m,p} B_m(q) \right]^2 , \tag{118}$$

where the weights $w(q,p)$ are proportional to enrollment mass $\chi(q,p)$.

9. Compute the sup norm distance between the updated and current spline coefficients:

$$\Delta = \max_p \max_{m=1,\ldots,M} \left| \theta_{m,p}^{\text{new}} - \theta_{m,p} \right| . \tag{119}$$

10. If Δ exceeds a chosen tolerance threshold, optionally damp the coefficient update

$$\theta_p \leftarrow \lambda \theta_p^{\text{new}} + (1-\lambda)\theta_p, \qquad \lambda \in (0,1], \tag{120}$$

and return to Step 2(b). Otherwise, the algorithm has converged.

Remark. The continuous-quality implementation follows the same logic as the discrete one. Given a candidate discount schedule $d(q,p)$, the college-side optimality conditions determine instructional spending $i(q,p)$, the college-side target for average student ability $\bar{e}(q,p)$, and the affine ability-specific tuition schedule $v(q,p,e)$. The household block then determines the allocation of students across college cells (q,p) on a dense quality grid, and the discount function adjusts until these objects are jointly consistent with market clearing in every college sub-market.

D.3 Solution Algorithm: Capital and Labor Market Equilibrium

The optimality condition of the firm with respect to capital $K(p)$ used in the p-specific output production is given by

$$\begin{aligned} r+\delta &= \Psi Y^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p)\left(\Psi K(p)\right)^{\rho(p)}+(1-\alpha(p))\left(\Gamma(p)L_t(p)\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}}\left(\Psi K(p)\right)^{\rho(p)-1} \\ &= \Psi Y^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p)+(1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}}\left(\Psi K(p)\right)^{\nu-1} \\ &= \Psi\left(\frac{Y}{\Psi K(p)}\right)^{1-\nu}\lambda(p)\alpha(p)\left(\alpha(p)+(1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}}, \end{aligned} \tag{121}$$

where $k(p)=\frac{\Psi K(p)}{\Gamma(p)L(p)}$ is the effective capital-labor ratio Similarly, the first order condition with respect to labor $L(p)$ is

$$\begin{aligned} w(p) &= \Gamma(p)Y^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p)\left(\Psi K(p)\right)^{\rho(p)}+(1-\alpha(p))\left(\Gamma(p)L(p)\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}}\left(\Gamma(p)L(p)\right)^{\rho(p)-1} \\ &= \Gamma(p)Y^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p)+(1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}}\left(\Psi K(p)\right)^{\nu-1+1-\rho(p)}\left(\Gamma(p)L(p)\right)^{\rho(p)-1} \\ &= \Gamma(p)\left(\frac{Y}{\Psi K(p)}\right)^{1-\nu}\lambda(p)(1-\alpha(p))\left(\alpha(p)+(1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} k(p)^{1-\rho(p)} \end{aligned} \tag{122}$$

Using (121) in the above equation we obtain:

$$w(p)=\frac{\Gamma(p)}{\Psi}\frac{1-\alpha(p)}{\alpha(p)}\cdot(r+\delta)\cdot k(p)^{1-\rho(p)} \tag{123}$$

which gives the capital intensity $k(p)$ as a function of p-specific wages and interest rates:

$$k(p) = \left(\frac{w(p)}{\frac{\Gamma(p)}{\Psi} \frac{1-\alpha(p)}{\alpha(p)} \cdot (r+\delta)} \right)^{\frac{1}{1-\rho(p)}} \tag{124}$$

This suggests the following algorithm:

1. Guess aggregate interest rate r, wage normalization factor $\Gamma(1)$ and p-specific wages $w(p)$ (for $p > 0$).

2. Solve the household problem (normalization the non-college wage to 1)

   - Compute aggregate assets K^s
   - Compute p-specific labor supply $\{L^s(p)\}_{p=0}^{P}$

3. Use eq. (123) to compute $k^d(p)$:

$$k^d(p) = \left(\frac{w(p)}{\frac{\Gamma(p)}{\Psi} \frac{1-\alpha(p)}{\alpha(p)} \cdot (r+\delta)} \right)^{\frac{1}{1-\rho(p)}} \tag{125}$$

   where $\Gamma(p) = \Gamma(1)\tilde{\Gamma}(p)$ with $\tilde{\Gamma}(p)$ being exogenous scaling parameters.

4. Compute aggregate capital demand: $K^d = \sum_p K^d(p) = \frac{\sum_p k^d(p) L^s(p) \Gamma(p)}{\Psi}$

5. Compute wage normalization factor $\Gamma(1)$:

$$\Gamma(1) = \frac{1}{\left(\frac{Y}{\Psi K^d(0)}\right)^{1-\nu} \lambda(0)(1-\alpha(0)) \left(\alpha(0) + (1-\alpha(0)) \left(\frac{1}{k^d(0)}\right)^{\rho(0)}\right)^{\frac{\nu-\rho(0)}{\rho(0)}} k^d(0)^{\rho(0)-1}} \tag{126}$$

6. For $p > 0$, compute p-specific wages consistent with p-specific household labor supply $L^s(p)$:

$$\tilde{w}(p) = \Gamma(p) \left(\frac{Y}{\Psi K^d(p)}\right)^{1-\nu} \lambda(p)(1-\alpha(p)) \left(\alpha(p) + (1-\alpha(p)) \left(\frac{1}{k^d(p)}\right)^{\rho(p)}\right)^{\frac{\nu-\rho(p)}{\rho(p)}} k^d(p)^{\rho(p)-1} \tag{127}$$

   Observe that both aggregate output Y and p-specific capital demand $K(p)$ are a function of p-specific labor supply by households $L^s(p)$.

7. Iterate on r and $w(p)$ for $p > 0$ and $\Gamma(1)$ until convergence, ensuring that the aggregate capital market clearing condition and p-specific labor market clearing conditions hold.

Special Case of Perfect Substitutes. With $\nu = 1$, equation (121) simplifies to

$$r + \delta = \Psi\lambda(p)\alpha(p)\left(\alpha(p) + (1-\alpha(p))\left(\frac{1}{k(p)}\right)^{\rho(p)}\right)^{\frac{1-\rho(p)}{\rho(p)}} \tag{128}$$

from which we can pin down in closed form $k(p)$ in each p as

$$k(p) = \left[\frac{1-\alpha(p)}{\left(\frac{r+\delta}{\Psi\lambda(p)\alpha(p)}\right)^{\frac{\rho(p)}{1-\rho(p)}} - \alpha(p)}\right]^{\frac{1}{\rho(p)}}. \tag{129}$$

Likewise, (122) simplifies to

$$w(p) = \Gamma(p)\lambda(p)(1-\alpha(p))\left(\alpha(p)k(p)^{\rho(p)} + (1-\alpha(p))\right)^{\frac{1-\rho(p)}{\rho(p)}}. \tag{130}$$

and we may now use (129) in (130) to see how wages are pinned down from $r + \delta$.